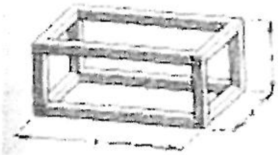
*AAT

Chapter 4: Test Review Packet

Name: Key

Date: ______________ Period: ______________

1. The frame for a shipping crate is to be constructed from 20 feet of 2 x 2 lumber. If the crate is to have square ends of side *x* feet, express the volume of the crate as a function of *x*.



$P = 8x + 4y = 20$

$\frac{4y}{4} = \frac{20 - 8x}{4}$

$y = 5 - 2x$

$V = x^2 y$

$= x^2(5 - 2x)$

$V = 5x^2 - 2x^3$

2. Find the quotient and remainder if f(x) is divided by p(x).

$f(x) = 5x + 9$; $p(x) = 3x^2 - x - 2$

$\frac{5x+9}{3x^2-x-2}$

Q: 0

R: $5x+9$

3. Find the quotient and remainder if f(x) is divided by p(x).

$f(x) = 7x^3 - 5x^2 + 2x - 5$; $p(x) = x - 2$

$$\begin{array}{r|l} & 7x^2 + 9x + 20 \\ x-2 & 7x^3 - 5x^2 + 2x - 5 \\ & -(7x^3 - 14x^2) \\ & 9x^2 + 2x \\ & -(9x^2 - 18x) \\ & 20x - 5 \\ & -(20x - 40) \\ & 35 \end{array}$$

Q: $7x^2 + 9x + 20$

R: 35

4. Find a polynomial with leading coefficient 1 and having the degree 4 and zeros -2, ±3, 8.

$f(x) = 1(x+2)(x+3)(x-3)(x-8)$

$= (x^2-9)(x^2-6x-16)$

$= x^4 - 6x^3 - 16x^2 - 9x^2 + 54x + 144 = x^4 - 6x^3 - 25x^2 + 54x + 144$

5. Use synthetic division to find the quotient and remainder if the first polynomial is divided by the second.

$24x^4 - 18x^2 + 3$; $x - \frac{1}{2}$

$$\begin{array}{r|rrrr|r} \frac{1}{2} & 24 & 0 & -18 & 0 & 3 \\ & & 12 & 6 & -6 & -3 \\ \hline & 24 & 12 & -12 & -6 & 0 \end{array}$$

Q: $24x^3 + 12x^2 - 12x - 6$

R: 0

6. Use synthetic division to find f(c).

$f(x) = 9x^3 + 6x^2 - 4x + 1$; $c = 3$

$$\begin{array}{r|rrrr} 3 & 9 & 6 & -4 & 1 \\ & & 27 & 99 & 285 \\ \hline & 9 & 33 & 95 & 286 \end{array}$$

7. Use synthetic division to find f(c). Leave your answer in radical form.

$f(x) = x^2 + 3x - 5$; $c = 2 + \sqrt{3}$

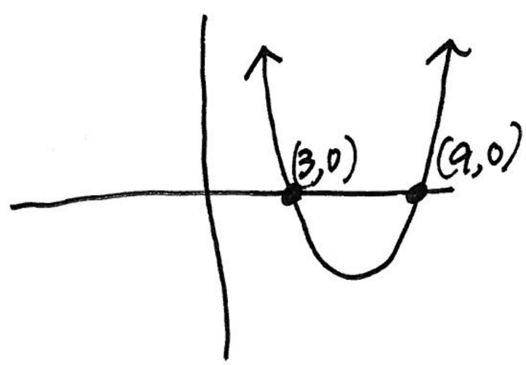
$$\begin{array}{r|rrr} 2+\sqrt{3} & 1 & 3 & -5 \\ & & 2+\sqrt{3} & 13+7\sqrt{3} \\ \hline & 1 & 5+\sqrt{3} & 8+7\sqrt{3} \end{array}$$

8. Find all values of k such that f(x) is divisible by the given linear polynomial.

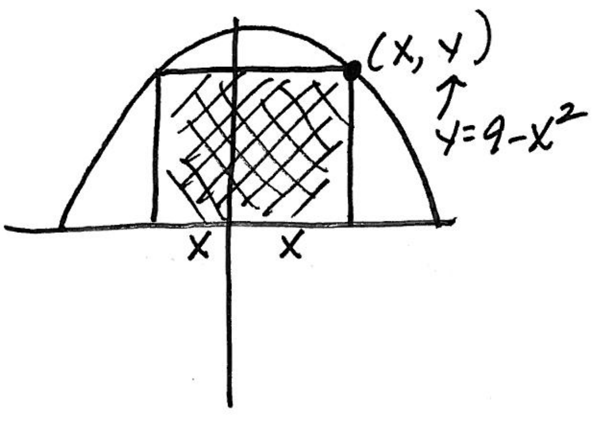
$f(x) = k^2x^3 - 108kx + 729$; x-3

$0 = 27k^2 - 324k + 729$

$k = 9$; $k = 3$



9. An arch has the shape of the parabola $y = 9 - x^2$. A rectangle is fit under the arch by selecting a point (x,y) on the parabola. If x = 1, the rectangle has base 2 and height 8. Find the base of a second rectangle that has the same area.



Area of New Rectangle:

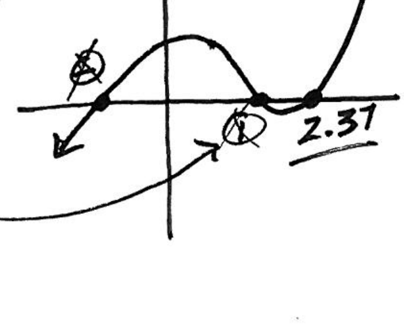
$A = l \cdot w$

$16 = y(2x)$

$16 = (9-x^2)(2x)$

$16 = 18x - 2x^3$

$2x^3 - 18x + 16 = 0$



$\begin{array}{r} 2.37 \\ \times\ 2 \\ \hline 4.74 \end{array}$

10. Show that the number is a zero of f(x) of the given multiplicity and express f(x) as a product of linear factors.

$f(x) = x^4 - 13x^3 + 57x^2 - 95x + 50$; 5, (multiplicity 2)

5	1	-13	57	-95	50
		5	-40	85	-50
5	1	-8	17	-10	0
		5	-15	10	
	1	-3	2	0	

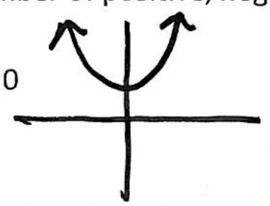
$f(x) = (x-5)^2(x-2)(x-1)$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1)$

11. Determine the number of positive, negative, and imaginary solutions of the equation.

$3x^4 + 4x^3 - 5x + 7 = 0$



0 pos; 0 neg; 4 imag.

12. A polynomial f(x) with real coefficients and leading coefficient 1 has the given zeros and degree. Express f(x) as a product of linear and quadratic polynomials with real coefficients that are irreducible over R.

9+4i, -1 + i; degree 4

$(x-(9+4i))(x-(9-4i))(x-(-1+i))(x-(-1-i))$

$= (x-9-4i)(x-9+4i)(x+1-i)(x+1+i)$

$= x^2 - 9x + \cancel{4ix} - 9x + 81 - \cancel{36i} - \cancel{4ix} + \cancel{36i} - 16i^2$

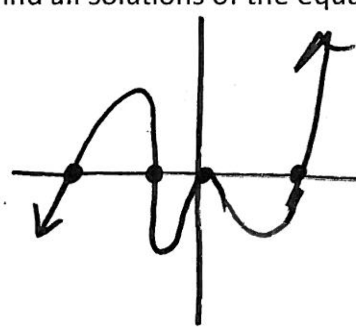
$= (x^2 - 18x + 97)$

$x^2 + x + x + 1 - i^2$

$(x^2 + 2x + 2)$

$f(x) = (x^2 - 18x + 97)(x^2 + 2x + 2)$

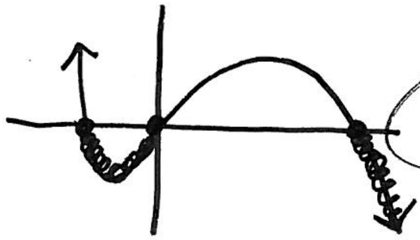
13. Find all solutions of the equation $20x^5 + 111x^4 + 51x^3 - 20x^2 = 0$.



$x = \left(-5, -\frac{4}{5}, 0\,(\text{d.r.}), \frac{1}{4}\right)$

14. Find all values of x such that f(x) < 0.

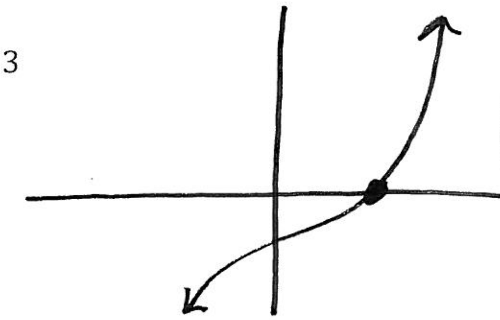
$f(x) = 4x - x^3$



$-2 < x < 0$ or $x > 2$

15. Find all values of x such that f(x) < 0.

$f(x) = \frac{1}{9}x^3 - 3$



$x < 3$

or

$(-\infty, 3)$

16. Find an equation of a rational function f that satisfies the conditions:

vertical asymptotes: x = -8, x = 4
horizontal asymptotes: y = 0
x-intercept: -2; f(0)= -10
hole at x = 4

$$\frac{a(x+2)(x-4)}{(x+8)(x-4)(x-4)}$$

$$f(0) = \frac{a(0+2)(0-4)}{(0+8)(0-4)(0-4)} = \frac{-8a}{128} = -10 \; ; \; a = 160$$

$$\frac{160(x^2-2x-8)}{(x+8)(x^2-8x+16)}$$

$$\frac{160x^2-320x-1280}{x^3-48x+128}$$

17. Find an equation of a rational function f that satisfies the conditions:

vertical asymptotes: x = -2, x = 4
horizontal asymptotes: y = 2
x-intercepts: -10, 5
hole at x = 0

$$\frac{2(x+10)(x-5)(x)}{1(x+2)(x-4)(x)}$$

$$\frac{2x(x^2+5x-50)}{x(x^2-2x-8)} = \frac{2x^3+10x^2-100x}{x^3-2x^2-8x}$$