*AAT

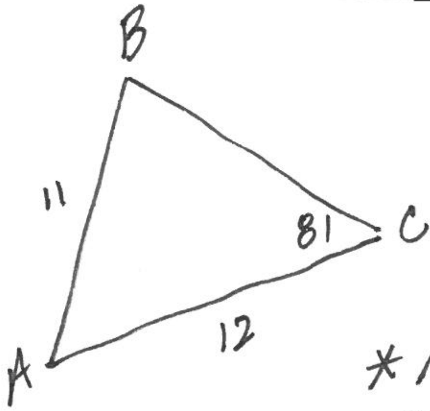
Name: Key

Chapter 8: Review for Test

Date: ________ Period: ________

1. Solve triangle ABC.

$\gamma = 81°$, $c = 11$, $b = 12$.



$$\frac{\sin 81}{11} = \frac{\sin B}{12}$$

$B = \sin^{-1}(1.08)$

* No Δ exists b/c sine of ∠ cannot be > 1.

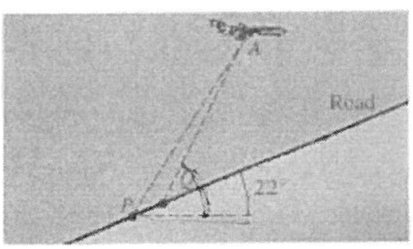
α = ________

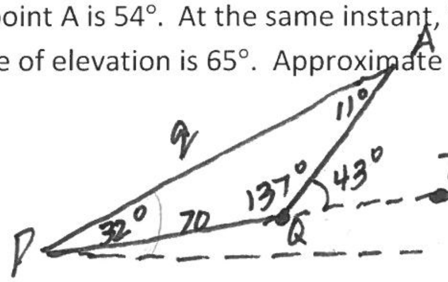
β = ________

a = ________

no Δ exists

2. A straight road makes an angle of 22° with the horizontal. Form a certain point P on the road, the angle of elevation of an airplane at point A is 54°. At the same instant, from another point Q, 70 meters farther up the road, the angle of elevation is 65°. Approximate the distance from P to the airplane.





$\angle APQ = 54 - 22 = 32°$

$\angle AQT = 65 - 22 = 43°$

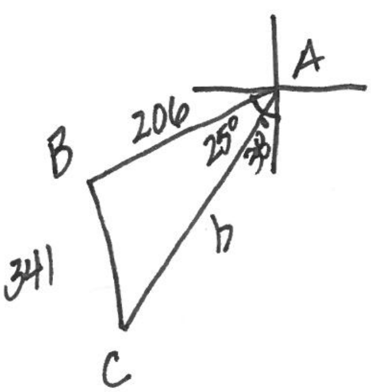
$\angle AQP = 180 - 43 = 137°$

$\angle PAQ = 180 - 137 - 32 = 11°$

$$\frac{\sin 11}{70} = \frac{\sin 137}{q}$$

$q \approx 250.2$ or $250\ m$

3. A surveyor notes that the direction from point A to point B is S63°W and the direction from A to point C is S38°W. The distance from A to B is 206 yards, and the distance from B to C is 341 yards. Approximate the distance from A to C.



$63 - 38 = 25°$

$$\frac{\sin 25}{341} = \frac{\sin C}{206}$$

$\angle C \approx 14.8°$

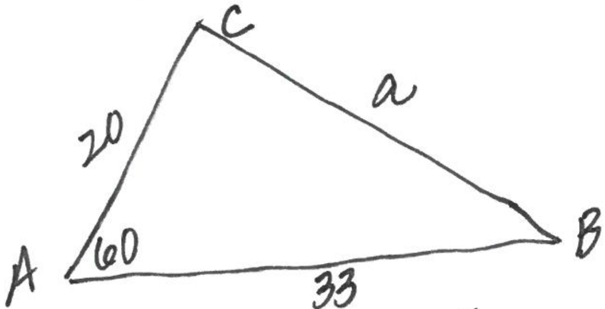
$\angle B = 180 - 25 - 15 = 140°$

$$\frac{\sin 25}{341} = \frac{\sin 140}{b}$$

$b \approx 518$ yds.

4. Solve triangle ABC.

$\alpha = 60°$, $b = 20$, $c = 33$.



$a=\sqrt{33^2+20^2-2(33)(20)\cos 60}$

$a\approx 28.79$ or $\underline{\underline{29}}$

$\frac{\sin B}{20}=\frac{\sin 60}{29}$

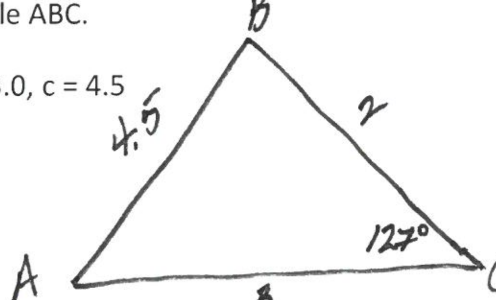
$\angle B\approx \underline{\underline{37°}}$

$\angle C=180°-60°-37°=$

$\angle C=\underline{\underline{83°}}$

5. Solve triangle ABC.

$a = 2.0$, $b = 3.0$, $c = 4.5$



$\frac{\sin 127}{4.5}=\frac{\sin A}{2}$

$\angle A\approx \underline{\underline{21°}}$

$4.5^2=2^2+3^2-2(2)(3)\cos C$

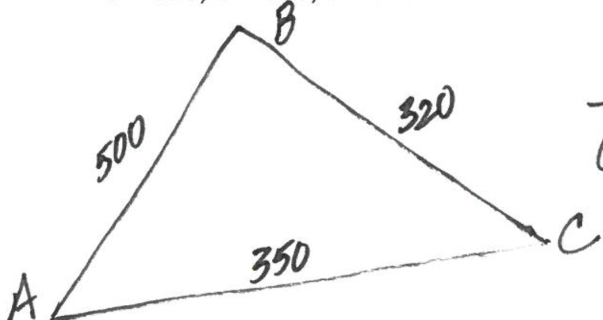
$\frac{4.5^2-2^2-3^2}{(-2\cdot 2\cdot 3)}=\cos C$

$\angle C\approx \underline{127°}$

$\angle B=180-127-21=\underline{32°}$

6. A triangular field has sides of lengths a, b, and c (in yards). Approximate the number of acres in the field (1 acre = 4840 yd^2).

$a = 320$, $b = 350$, $c = 500$



$500^2=320^2+350^2-2(320)(350)\cos C$

$\frac{500^2-320^2-350^2}{(-2\cdot 320\cdot 350)}=\cos C$

$\angle C\approx \underline{96.43°}$

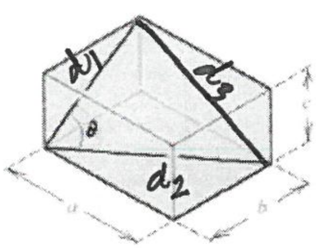
$A=\frac{1}{2}ab\sin\gamma$

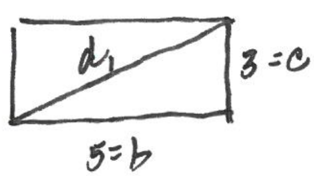
$=\frac{1}{2}(320)(350)\sin 96$

$=55693.223\div 4840$

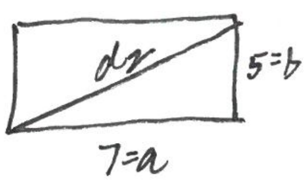
$=11.5$ acres

7. The rectangular box shown below in the figure has dimensions a x b x c. Suppose that a = 7", b = 5", and c = 3". Approximate the angle θ formed by a diagonal of the base and a diagonal of the 5" x 3" side.

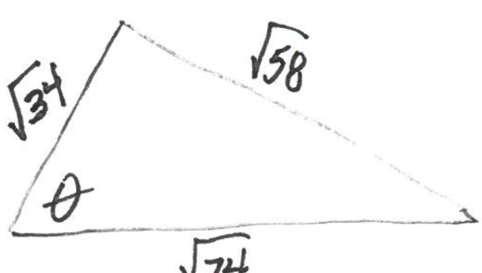


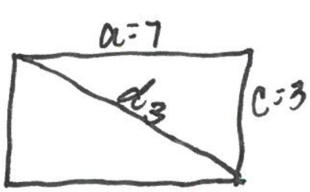


$d_1=\sqrt{9+25}=\sqrt{34}$



$d_2=\sqrt{25+49}=\sqrt{74}$

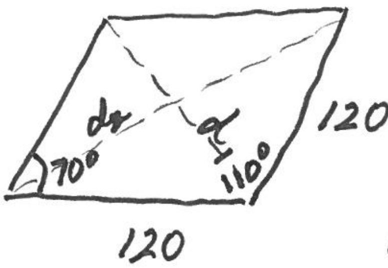




$d_3=\sqrt{49+9}=\sqrt{58}$

$58=34+74-2(\sqrt{34})(\sqrt{74})\cos\theta$; $\frac{(58-34-74)}{(-2\cdot\sqrt{34}\cdot\sqrt{74})}=\cos\theta$; $\theta\approx 60.11°$

8. A rhombus has sides of length 120 cm, and the angle at one of the vertices is 70°. Approximate the lengths of the diagonals to the nearest tenth of a cm.



$$d_1 = \sqrt{120^2 + 120^2 - 2(120)(120)\cos 70°}$$

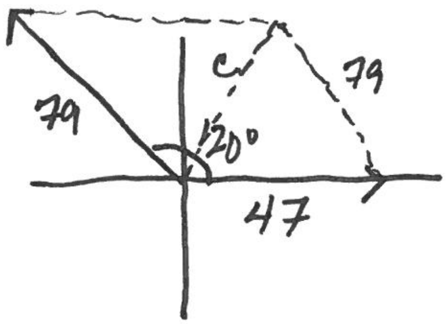
$$d_1 = 137.7\text{ cm.}$$

$$d_2 = \sqrt{120^2 + 120^2 - 2(120)(120)\cos 110°}$$

$$d_2 = 196.6\text{ cm.}$$

9. The vectors **a** and **b** represent two forces acting at the same point, and θ is the smallest positive angle between **a** and **b**. Approximate the magnitude of the resultant force.

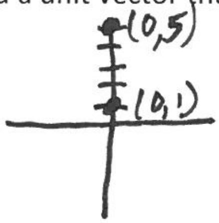
$||a|| = 47$ lbs, $||b|| = 79$ lbs, $\theta = 120°$



$$c^2 = 79^2 + 47^2 - 2(79)(47)\cos 60$$

$c \approx 68.8$ or 69 lb.

10. Find a unit vector that has the same direction as the vector **a** = (0, 5).



(0, 1)

11. Find the angle between the two vectors <-2, 6> and <-8, 9>.

$$\cos\theta = \frac{a \cdot b}{||a|| \cdot ||b||} = \frac{(-2 \cdot -8) + (6 \cdot 9)}{(\sqrt{4+36})(\sqrt{64+81})} = \frac{70}{\sqrt{40} \cdot \sqrt{145}} = \frac{70}{\sqrt{5800}}$$

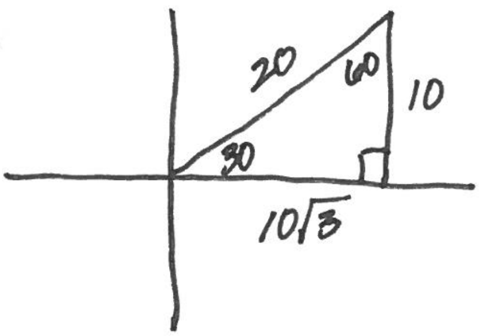
$$\theta = \cos^{-1}\left(\frac{70}{\sqrt{5800}}\right)$$

$$\theta \approx 23.2°$$

12. Find i^{536}

$$(i^2)^{268} = (-1)^{268} = 1$$

13. Express the complex number $10\sqrt{3} + 10i$ in trig form with $0 \le \theta \le 2\pi$.



$$r = \sqrt{10^2 + (10\sqrt{3})^2} = \sqrt{400} = 20$$

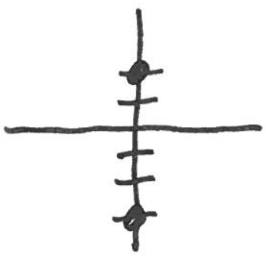
$$\theta = \sin^{-1}\left(\frac{10}{20}\right) = \frac{\pi}{6} \text{ or } 30^\circ$$

$$z = 20\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

14. Use trig forms to find $z_1 z_2$ and $\frac{z_1}{z_2}$.

$0+2i \quad 0-3i$

$z_1 = 2i$, $z_2 = -3i$



$$z_1 = 2\,\text{cis}\,\frac{\pi}{2} \qquad z_2 = 3\,\text{cis}\,\frac{3\pi}{2}$$

$$z_1 z_2 = 2 \cdot 3\,\text{cis}\left(\frac{\pi}{2} + \frac{3\pi}{2}\right)$$

$$= 6\,\text{cis}\,\frac{4\pi}{2} = 6\,\text{cis}\,2\pi$$

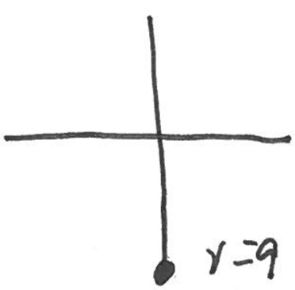
$$= 6(\cos 2\pi + i\sin 2\pi)$$

$$= 6(1 + 0i) = 6$$

$$\frac{z_1}{z_2} = \frac{2}{3}\,\text{cis}\left(\frac{\pi}{2} - \frac{3\pi}{2}\right) = \frac{2}{3}(\cos -\pi + i\sin -\pi)$$

$$= \frac{2}{3}(-1 + 0i) = -\frac{2}{3}$$

15. Find the two square roots of $-9i$. $0-9i$



$r = 9$

$\theta = \frac{3\pi}{2}$

$z = 9\,\text{cis}\,\frac{3\pi}{2}$

$$W_k = \sqrt{9}\left[\cos\left(\frac{\frac{3\pi}{2} + 2\pi k}{2}\right) + i\sin\left(\frac{\frac{3\pi}{2} + 2\pi k}{2}\right)\right]$$

$$W_0 = 3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

$$= 3\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = -\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$$

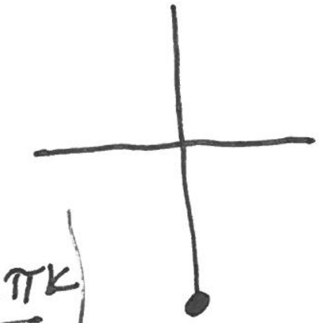
$$W_1 = 3\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

$$= 3\left(\frac{\sqrt{2}}{2} + -\frac{\sqrt{2}}{2}i\right) = \frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i$$

16. Find the solutions of the equation $x^3 + 343i = 0$.

$x^3 = -343i$

$0 - 343i$



$z = 343 \text{ cis } \frac{3\pi}{2}$

$W_k = \sqrt[3]{343}\left(\cos \frac{\frac{3\pi}{2} + 2\pi k}{3} + i \sin \frac{\frac{3\pi}{2} + 2\pi k}{3}\right)$

$W_0 = 7(\cos 90 + i \sin 90)$

$= 7(0 + i) = 7i$

$W_1 = 7(\cos 210 + i \sin 210)$

$= 7\left(-\frac{\sqrt{3}}{2} + -\frac{1}{2}i\right) = -\frac{7\sqrt{3}}{2} - \frac{7}{2}i$

$W_2 = 7(\cos 330 + i \sin 330)$

$7\left(\frac{\sqrt{3}}{2} + -\frac{1}{2}i\right) = \frac{7\sqrt{3}}{2} - \frac{7}{2}i$

Helpful Hints:

$a \cdot b = ||a||\,||b|| \cos \theta$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)]$

$\text{comp}_b a = \frac{a \cdot b}{||b||}$

$w_k = \sqrt[n]{r}\,[\cos(\frac{\theta + 2\pi k}{n}) + i \sin(\frac{\theta + 2\pi k}{n})]$

$z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$

$[r(\cos\theta + i \sin\theta)]^n = r^n(\cos n \cdot \theta + i \sin n \cdot \theta)$