

## Openers #7

Name: Key

Each day when you come into class, there will be a problem projected for you to complete. Find the appropriate box to complete the problem in and work on it when you arrive.

Date: ____ / ____ / ____	7-1  Verify the identities.  1. $(\tan u + \cot u)(\cos u + \sin u) = \csc u + \sec u$ $\left(\frac{\sin u}{\cos u} + \frac{\cos u}{\sin u}\right)(\cos u + \sin u) = \frac{\sin^2 u + \cos^2 u}{\cos u - \sin u} (\csc u + \sec u)$ $= \frac{1}{\cos u \sin u} (\cos u + \sin u) = \frac{\cos u}{\cos u \sin u} + \frac{\sin u}{\cos u \sin u}$ $= \frac{1}{\sin u} + \frac{1}{\cos u} \in \csc u + \sec u$ 2. $\tan^2 \alpha - \sin^2 \alpha = \tan^2 \alpha \sin^2 \alpha$ $\frac{\sin^2 \alpha}{\cos^2 \alpha} - \sin^2 \alpha = \frac{\sin^2 \alpha - \sin^2 \alpha \cos^2 \alpha}{\cos^2 \alpha}$ $= \frac{\sin^2 \alpha (1 - \cos^2 \alpha)}{\cos^2 \alpha}$ $= \frac{\sin^2 \alpha}{\cos^2 \alpha} \cdot \sin^2 \alpha = \tan^2 \alpha \cdot \sin^2 \alpha$
Date: ____ / ____ / ____	7-2-1  Find all solutions of the equations.  1. $\csc y = \sqrt{2}$ $\sin y = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ ; $\frac{\pi}{4} + 2\pi n; \frac{3\pi}{4} + 2\pi n$  2. $\cot \theta + 1 = 0$ $\cot \theta = -1$ $\frac{3\pi}{4} + 2\pi n; \frac{7\pi}{4} + 2\pi n$ - or - $\frac{3\pi}{4} + \pi n$  3. $4\sin^2 x - 3 = 0$ $\frac{4\sin^2 x}{4} = \frac{3}{4}$ ; $\sin^2 x = \frac{3}{4}$ ; $\sin x = \pm \frac{\sqrt{3}}{2}$ $\frac{\pi}{3} + \pi n; \frac{2\pi}{3} + \pi n$ - or - $\frac{\pi}{3} + 2\pi n; \frac{2\pi}{3} + 2\pi n;$ $\frac{4\pi}{3} + 2\pi n; \frac{5\pi}{3} + 2\pi n$

Date:

7-2-2

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Find the solutions that are in the interval  $[0, 2\pi)$ 

1.  $\cot^2 \theta - \cot \theta = 0$

$$\cot \theta (\cot \theta - 1) = 0$$

$$\cot \theta = 0$$

$$\frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cot \theta = 1$$

$$\frac{\pi}{4}, \frac{5\pi}{4}$$

2.  $2\cos^2 t + 3\cos t + 1 = 0$

$$(2\cos t + 1)(\cos t + 1) = 0$$

$$2\cos t + 1 = 0 \quad \cos t + 1 = 0$$

$$\cos t = -\frac{1}{2}$$

$$\frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\cos t = -1$$

$$\pi$$

Date:

7-3

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Express  $\sin 89^\circ 41'$  as a cofunction of a complementary angle.

$$90^\circ - 89^\circ 41' = \cos 0^\circ 19'$$

Find the exact values.

a)  $\sin \frac{2\pi}{3} + \sin \frac{\pi}{4}$

$$\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} = \frac{\sqrt{3} + \sqrt{2}}{2}$$

$$\sin(\alpha + \beta)$$

b)  $\sin \frac{11\pi}{12}$  (use  $\frac{2\pi}{3} + \frac{\pi}{4}$ )

$$\sin \frac{2\pi}{3} \cos \frac{\pi}{4} + \cos \frac{2\pi}{3} \sin \frac{\pi}{4}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + -\frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \text{ or } -\frac{\sqrt{6}-\sqrt{2}}{4}$$

Express as a trig function.

a)  $\cos 13^\circ \cos 50^\circ - \sin 13^\circ \sin 50^\circ$

$$\cos(\alpha + \beta) = \cos(13 + 50)$$

$$\cos 63^\circ$$

$$\begin{array}{c} 13 \\ \backslash \\ \alpha \\ \square \\ 5 \end{array} \quad \begin{array}{c} 12 \\ \backslash \\ \beta \\ \square \\ 4 \end{array} \quad \sin(\alpha + \beta)$$

Verify the reduction formula.

$$\sin(x + \frac{\pi}{2}) = \cos x$$

$$\sin x \cos \frac{\pi}{2} + \cos x \sin \frac{\pi}{2}$$

$$\sin x \cdot (0) + \cos x \cdot (1)$$

$$0 + \cos x$$

$$\cos x$$

$$\sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$(\frac{12}{13})(\frac{4}{5}) + (\frac{5}{13})(\frac{3}{5})$$

$$= \frac{48}{65} + \frac{15}{65}$$

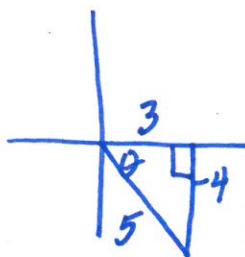
$$= \frac{63}{65}$$

Date:

7-4

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Find the exact values of  $\sin 2\theta$ ,  $\cos 2\theta$ , and  $\tan 2\theta$  for  $\sin \theta = -\frac{4}{5}$ ;  $270^\circ < \theta < 360^\circ$



$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ &= 2 \left(-\frac{4}{5}\right) \left(\frac{3}{5}\right) = \frac{-24}{25}\end{aligned}$$

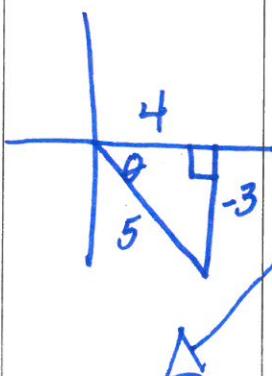
$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\begin{aligned}&= \frac{2 \left(-\frac{4}{3}\right)}{1 - \left(-\frac{4}{3}\right)^2} = \frac{-\frac{8}{3}}{1 - \frac{16}{9}} \\ &= \frac{-\frac{8}{3}}{-\frac{7}{9}} = \frac{8}{3} \cdot \frac{3}{7} = \frac{24}{7}\end{aligned}$$

$$\begin{aligned}\cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= \left(\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2 \\ &= \frac{9}{25} - \frac{16}{25} = \frac{-7}{25}\end{aligned}$$

$$\begin{aligned}&= \frac{-\frac{8}{3}}{\frac{-7}{9}} = \frac{8}{3} \cdot \frac{9}{7} = \frac{24}{7}\end{aligned}$$

Find the exact values of  $\sin \theta/2$ ,  $\cos \theta/2$ , and  $\tan \theta/2$  for  $\csc \theta = -\frac{5}{3}$ ;  $-90^\circ < \theta < 0^\circ$



$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$= \pm \sqrt{\frac{1 - \frac{4}{5}}{2}} = \pm \sqrt{\frac{\frac{1}{5}}{2}} = \pm \sqrt{\frac{1}{10}} = \pm \frac{1}{\sqrt{10}} = \pm \frac{\sqrt{10}}{10}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}}$$

$$= \sqrt{\frac{1 + \frac{4}{5}}{2}} = \sqrt{\frac{\frac{9}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{3\sqrt{10}}{10}$$

Use the half-angle formula to find the exact values of  $\cos 165^\circ$ .

$$\begin{aligned}\tan \frac{\theta}{2} &= \frac{\sin \theta}{1 + \cos \theta} \\ &= \frac{-\frac{3}{5}}{1 + \frac{4}{5}} = \frac{-\frac{3}{5}}{\frac{9}{5}} = -\frac{1}{3}\end{aligned}$$

$$\cos \frac{330}{2} = -\sqrt{\frac{1 + \cos 330}{2}} = -\sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2+\sqrt{3}}{4}} = -\frac{\sqrt{2+\sqrt{3}}}{2}$$

Find the solutions of  $\cos t - \sin 2t = 0$  that are in the interval  $[0, 2\pi)$ .

$$\cos t - (2 \sin t \cos t) = 0$$

$$\cos t(1 - 2 \sin t) = 0$$

$$\cos t = 0$$

$$\frac{\pi}{2}, \frac{3\pi}{2}$$

$$1 - 2 \sin t = 0$$

$$\sin t = \frac{1}{2}; \frac{\pi}{6}, \frac{5\pi}{6}$$

Date:

7-5

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Express  $5 \cos u \cos 5u$  as a sum or difference.

$$\frac{5}{2} (\cos(u+5u) + \cos(u-5u)) = \frac{5}{2} (\cos 6u + \cos 4u)$$

$$\text{or}$$

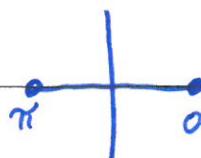
$$\frac{5}{2} \cos 6u + \frac{5}{2} \cos 4u$$

Express  $\cos 5t + \cos 6t$  as a product.

$$2 \cos \frac{5t+6t}{2} \cos \frac{5t-6t}{2} = \cos 4t$$

$$2 \cos \frac{11}{2}t \cos \frac{1}{2}t$$

7-5 continued...



$$\text{or } \frac{2t}{2} = \frac{0+2\pi n}{2} \quad \frac{2t}{2} = \frac{\pi + 2\pi n}{2}$$

$$t = \pi n \quad \text{or} \quad t = \frac{\pi}{2} + \pi n$$

Use sum-to-product formulas to find the solutions of the equations.

1)  $\sin t + \sin 3t = \sin 2t$

$2 \sin \frac{t+3t}{2} \cos \frac{t-3t}{2} = \sin 2t$

$2 \sin 2t \cos t = \sin 2t$

$2 \sin 2t \cos t - \sin 2t = 0$

$\sin 2t(2 \cos t - 1) = 0$

$\sin 2t = 0$

$2 \cos t - 1 = 0$

$\frac{2t}{2} = \frac{\pi n}{2}$

$\cos t = \frac{1}{2}$

$t = \frac{\pi}{3} + 2\pi n$

$t = \frac{5\pi}{3} + 2\pi n$

2)  $\cos 4x - \cos 3x = 0$

$-2 \sin \frac{4x+3x}{2} \sin \frac{4x-3x}{2} = 0$

$-2 \left( \sin \frac{7x}{2} \right) \left( \sin \frac{x}{2} \right) = 0$

$\frac{7x}{2} = \pi n$

$\sin \frac{7x}{2} = 0$

$\sin \frac{x}{2} = 0$

$\frac{7x}{2} = \pi n$

$\frac{x}{2} = \pi n$

$x = \frac{2\pi n}{7}$

$x = 2\pi n$

(included in)

Date:

7-6

$n = 7$

$x = 2\pi$

Find the exact value of the expression.

a)  $\sin^{-1} \left( -\frac{1}{2} \right)$

$\frac{-\pi}{6}$

b)  $\arcsin 0$

0

c)  $\arctan(\tan \frac{\pi}{4})$

$\frac{\pi}{4}$

d)  $\tan(\cos^{-1} 0)$

undefined

e)  $\sin(\tan^{-1} \sqrt{3})$

$\frac{\sqrt{3}}{2}$

f)  $\sin(2\tan^{-1} \frac{5}{12})$

$2 \sin A \cos A$

$2 \left( \frac{5}{13} \right) \left( \frac{12}{13} \right) = \frac{120}{169}$

g)  $\cos(\sin^{-1} \frac{4}{5} + \tan^{-1} \frac{3}{4})$

$\frac{1}{5} + \frac{3}{4}$

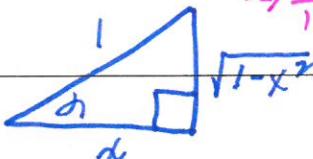
h)  $\sec(\tan^{-1} \frac{17}{4})$

$\sqrt{17}$

$\sqrt{17^2 + 4^2} = \sqrt{285}$

$\cos A \cos B - \sin A \sin B$

$\frac{3}{5} \cdot \frac{4}{5} - \frac{4}{5} \cdot \frac{3}{5} = \frac{12}{25} - \frac{12}{25} = 0$

Write the expression  $\tan(\arccos x)$  as an algebraic expression in x for  $x > 0$ .

$\tan \alpha = \frac{\sqrt{1-x^2}}{x}$

