Openers #4

Name: *Key*

Each day when you come into class, there will be a problem projected for you to complete. Find the appropriate box to complete the problem in and work on it when you arrive.

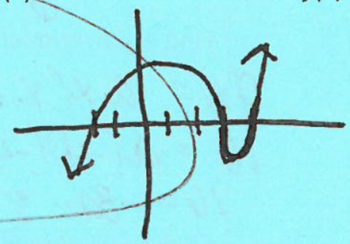
Date: ___/___/___

4-1

Find all values of $f(x) = \frac{1}{6}(x+2)(x-3)(x-4)$ such that $f(x) > 0$ and all x such that $f(x) < 0$.

$f(x) > 0$ if $(-2 < x < 3$ or $x > 4)$

$f(x) < 0$ if $(x < -2$ or $3 < x < 4)$



If $f(x) = kx^3 + x^2 - kx + 2$, find a number k such that the graph of f contains the point (2,12).

$12 = 2^3k + 2^2 - 2k + 2$

$12 = 8k + 4 - 2k + 2$

$12 = 6k + 6$

$6 = 6k$

$1 = k$

If one zero of $f(x) = x^3 - 3x^2 - kx + 12$ is -2, find two other zeros.

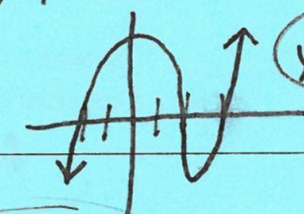
$f(-2) = (-2)^3 - 3(-2)^2 - (-2)k + 12$

$0 = -8 - \cancel{12} + 2k + \cancel{12}$

$\frac{8}{2} = \frac{2k}{2}$

$4 = k$

$y = x^3 - 3x^2 - 4x + 12$



$x = 2, 3$

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4-2

Find the quotient and remainder if $f(x)$ is divided by $p(x)$.

Q: $3x^2 + 2$

R: $-21x^2 + 5x - 9$

$f(x) = 3x^5 - 4x^3 + x + 5$; $p(x) = x^3 - 2x + 7$

$$
\begin{array}{r}
3x^2 \quad +2 \\
x^3 + 0x^2 - 2x + 7 \;\big)\; 3x^5 + 0x^4 - 4x^3 + 0x^2 + x + 5 \\
-\;\; 3x^5 + 0x^4 - 6x^3 + 21x^2 \qquad\qquad \\
\hline
2x^3 - 21x^2 + x + 5 \\
-\;\; 2x^3 + 0x^2 - 4x + 14 \\
\hline
-21x^2 + 5x - 9
\end{array}
$$

If $f(x) = -4x^4 + 3x^3 - 5x^2 + 7x - 10$, use the remainder theorem to find $f(-2)$.

$$
\begin{array}{r|rrrrr}
-2 & -4 & 3 & -5 & 7 & -10 \\
 & \downarrow & 8 & -22 & 54 & -122 \\
\hline
 & -4 & 11 & -27 & 61 & -132
\end{array}
$$

Use the factor theorem to show that x-3 is a factor of $f(x) = 2x^4 - 5x^3 - 4x^2 + 9$.

$2(3^4) - 5(3^3) - 4(3^2) + 9$

$162 - 135 - 36 + 9 = 0$

$f(3) = 0$

yes

Date:

___/___/___

4-3-1

Use synthetic division to find the quotient and remainder if f(x) is divided by p(x).

$f(x) = 6x^5 - 4x^2 + 8$; $p(x) = x+2$

Q: $6x^4 - 12x^3 + 24x^2 - 52x + 104$

R: -200

$$\begin{array}{r|rrrrrr} -2 & 6 & 0 & 0 & -4 & 0 & 8 \\ & & -12 & 24 & -48 & 104 & -208 \\ \hline & 6 & -12 & 24 & -52 & 104 & -200 \end{array}$$

Find a polynomial $f(x)$ of degree 3 that has the indicated zeros -3, -2, 0 and satisfies $f(-4) = 16$.

$f(x) = ax(x+3)(x+2)$

$16 = a(-4)(-4+3)(-4+2)$

$16 = -8a$; $a = -2$

$-2x(x^2+5x+6)$

$f(x) = -2x^3 - 10x^2 - 12x$

Find a polynomial $f(x)$ of degree 7 such that -2 and 2 are both zeros of multiplicity 2, 0 is a zero of multiplicity 3, and $f(-1) = 27$.

$x^3(x+2)^2(x-2)^2$

$a(-1)^3(-1+2)^2(-1-2)^2$

$-9a = 27$; $a = -3$

$-3x^3(x^2-4)(x^2-4)$

$-3x^3(x^4-8x^2+16)$

$-3x^7 + 24x^5 - 48x^3$

Find the zeros of $f(x) = x(x+1)^4(3x-7)^2$, and state the multiplicity of each zero.

-1 (mult. 4)

0 (mult. 1)

$\frac{7}{3}$ (mult. 2)

Date:

___/___/___

4-3-2

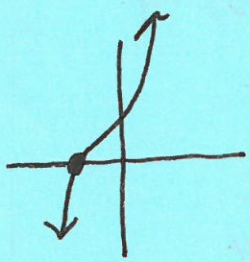
Show that the number is a zero of $f(x)$ of the given multiplicity, and express $f(x)$ as a product of linear factors. $f(x) = x^4 - 9x^3 + 22x^2 - 32$; 4(multiplicity 2)

$f(x) = (x-4)^2(x-2)(x+1)$

$$\begin{array}{r|rrrrr} 4 & 1 & -9 & 22 & 0 & -32 \\ & & 4 & -20 & 8 & 32 \\ \hline 4 & 1 & -5 & 2 & 8 & 0 \\ & & 4 & -4 & -8 & \\ \hline & 1 & -1 & -2 & 0 & \end{array}$$

$x^2 - x - 2$

$(x-2)(x+1)$

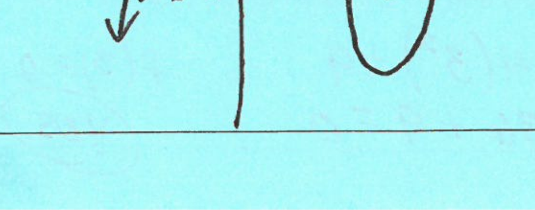


Determine the number of positive, negative, and nonreal solutions of $3x^3 - 4x^2 + 3x + 7 = 0$.

1 negative; 2 imaginary

The polynomial function $f(x) = x^5 - 2.5x^4 - 12.75x^3 + 19.625x^2 + 27.625x + 7.5$ has only real zeros. Use a graph to factor it.

$f(x) = (x+3)(x+.5)^2(x-2.5)(x-4)$



Date: ___/___/___	4-4

A polynomial of $f(x)$ with real coefficients and leading coefficient 1 has the given zeros and degree. Express $f(x)$ as a product of linear and quadratic polynomials with real coefficients that that are irreducible over $\mathbb{R}$. $-3, 1-7i$; degree 3

$(x+3)(x-(1-7i))(x-(1+7i))$

$(x+3)(x-1+7i)(x-1-7i)$

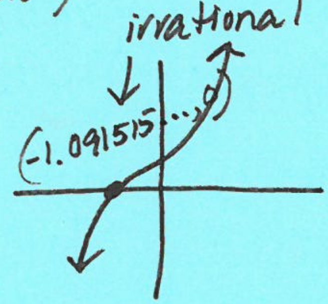
$(x+3)(x^2-x-\cancel{7ix}-x+1+\cancel{7i}+\cancel{7ix}-\cancel{7i}-49i^2)$

$(x+3)(x^2-2x+50)$

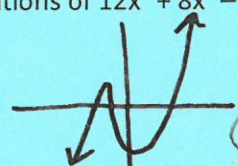
Show that the equation $2x^5+3x^3+7=0$ has no rational roots.

Possible Rational Roots $\rightarrow$ $\dfrac{\pm1, \pm7}{\pm1, \pm2} = \pm1, \pm\frac{1}{2}, \pm7, \pm\frac{7}{2}$

irrational



Find all solutions of $12x^3+8x^2-3x-2=0$.



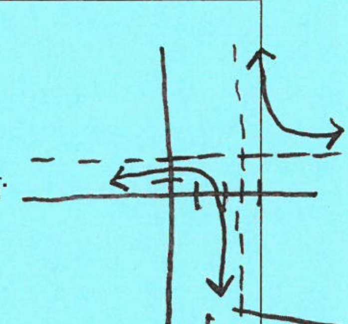
$-\frac{2}{3}, \pm\frac{1}{2}$

The polynomial function $f(x) = 0.5x^3 + .65x^2 - 5.365x + 1.5375$ has only real zeros. Use a graph to factor it.

$.5(x+4.1)(x-.3)(x-2.5)$

Date: ___/___/___	4-5

Label vertical and horizontal asymptotes and sketch the graph of $f(x) = \frac{4x}{2x-5}$.



Zero's of denominator $\rightarrow$ V. asymptote: $x = 2.5$

h. asymptote: $y = \frac{4}{2} = 2$

Simplify $f(x) = \frac{x^2-x-6}{x^2-2x-3}$. $\dfrac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+1)}$;

- if deg num < deg den: $y=0$ is horizontal asym.
- if num = den: y = leading coef
- if num > den: no horizontal asym

Find an equation of a rational function f that satisfies the given conditions:

vertical asymptotes: $x = -1$, $x = 3$

horizontal asymptote: $y = 2$; deg of num = deg of denom.

x-intercepts: $-2, 1$; hole at $x = 0$. $\dfrac{2x(x+2)(x-1)}{1x(x+1)(x-3)} = \dfrac{2x(x^2+x-2)}{1x(x^2-2x-3)} = \dfrac{2x^3+x^2-2x}{x^3-2x^2-3x}$

(numerator)

$f(x) = \dfrac{2(x^3+x^2-2x)}{1(x^3-2x^2-3x)}$

$*\ \dfrac{2x^3+2x^2-4x}{x^3-2x^2-3x}$