Openers #8

Name: Key

Each day when you come into class, there will be a problem projected for you to complete. Find the appropriate box to complete the problem in and work on it when you arrive.

<table>
<tr><td>Date:

___/___/___</td><td>8-1

Solve ΔABC. Solve

$\alpha = 103.45^\circ$, $\gamma = 27.19^\circ$, $b = 38.84$

$\beta = 180 - 103.45 - 27.19 = 49.36^\circ$

$\frac{\sin 103.45}{a} = \frac{\sin 49.36}{38.84}$

$a = 49.78$

$\frac{\sin 27.19}{c} = \frac{\sin 49.36}{38.84}$

$c = 23.39$

Solve ΔABC.

$\gamma = 73.01^\circ$, $a = 17.31$, $c = 20.24$

$\frac{\sin \alpha}{17.31} = \frac{\sin 73.01}{20.24}$

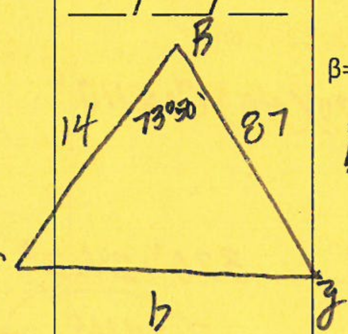
$\alpha = 54.88^\circ$

$\beta = 180 - 73.01 - 54.88 = 52.11^\circ$

$\frac{\sin 52.11}{b} = \frac{\sin 73.01}{20.24}$

$b = 16.70$</td></tr>
<tr><td>Date:

___/___/___

</td><td>8-2

1. Solve ΔABC.

$\beta = 73^\circ 50'$, $c = 14.0$, $a = 87.0$

$b^2 = 87^2 + 14^2 - 2(87)(14)\cos 73^\circ 50'$

$b = 84.2$

$\frac{\sin \gamma}{14} = \frac{\sin 73^\circ 50}{84.2}$

$\gamma = 9^\circ 11'$

$\alpha = 180 - 73^\circ 50' - 9^\circ 11' = 96^\circ 59'$

2. Solve ΔABC.

$a = 10$, $b = 15$, $c = 12$

$\frac{15^2 - 12^2 - 10^2}{(-2(12)(10))} = \cos B$

$85^\circ 30'$ (or 85°) $= \angle B$

$\frac{\sin 85}{15} = \frac{\sin A}{10}$

$\angle A = 41^\circ 40'$ or 42°

$\angle C = 180 - 85^\circ 30' - 41^\circ 40'$
$\angle C = 52^\circ 50'$ or 53°

3. Approximate the area of ΔABC. $\gamma = 45^\circ$, $b = 10.0$, $a = 15.0$

$A = \frac{1}{2} ab \sin \gamma$</td></tr>
</table>

$= \frac{1}{2}(15)(10)\sin 45^\circ$

≈ 53.0

Date:

___/___/___

8-3-1

If a = 2<5,-4> & b = -<6,0>, find 4a + 5b and ||a||.

$a = \langle 10,-8\rangle \quad b = \langle -6,0\rangle$

$4a = \langle 40,-32\rangle,\ 5b = \langle -30,0\rangle$

$4a+5b = \langle 10,-32\rangle$

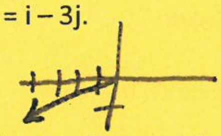
$\|a\| = \sqrt{10^2+(-8)^2}$

$= \sqrt{100+64}$

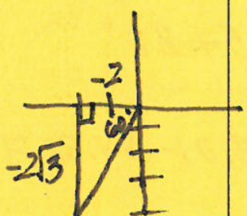
$= \sqrt{164} \approx 2\sqrt{41} \approx 12.8$

Sketch the vector a+b if a = -5i + 2j and b = i – 3j.

$a+b = -4i - j$



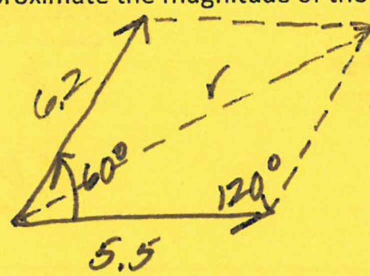
Find the magnitude of the vector **a** and the smallest positive angle θ from the positive x-axis if a=<-2, -2$\sqrt{3}$>



$\|a\| = \sqrt{(-2)^2+(-2\sqrt{3})^2} = \sqrt{16} = 4$

$\tan\theta = \dfrac{-2\sqrt{3}}{-2} = \sqrt{3}$; θ is in QIII $\Rightarrow \theta = 240°$ or $\dfrac{4\pi}{3}$

Approximate the magnitude of the resultant force between vector a=5.5 lb, b=6.2 lb, and θ=60°.



$\|r\| = \sqrt{5.5^2+6.2^2-2(5.5)(6.2)\cos 120°}$

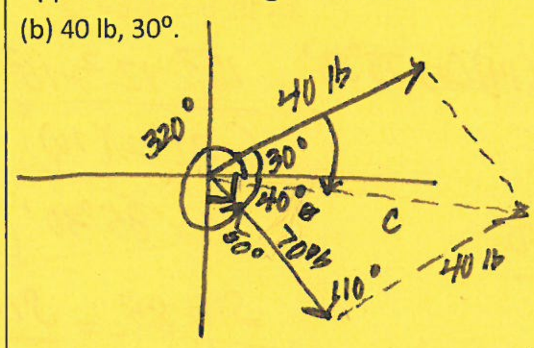
$= \sqrt{102.71} \approx 10.1$ lbs

Date:

___/___/___

8-3-2

Approximate the magnitude and direction of the resultant vector if (a) 70 lb, 320°, and (b) 40 lb, 30°.



$C^2 = 70^2+40^2-2(70)(40)\cos 110°$

$C \approx 91.7$

$\dfrac{\sin\theta}{40} = \dfrac{\sin 110}{91.7}$

$\theta = 24.2°$

$320° + 24.2° = 344°$

Find a unit vector in the same direction and opposite direction of a = 5i – 3j.

$\|a\| = \sqrt{5^2+(-3)^2} = \sqrt{34}$

$u = \dfrac{a}{\|a\|} = \dfrac{5}{\sqrt{34}}i - \dfrac{3}{\sqrt{34}}j$

$-u = -\dfrac{5}{\sqrt{34}}i + \dfrac{3}{\sqrt{34}}j$

Find the net force (sum of forces) and the additional force G such that an equilibrium occurs (F=0). $F_1 = \langle 4,3\rangle$, $F_2 = \langle -2,-3\rangle$, $F_3 = \langle 5,2\rangle$

$F_1+F_2+F_3 = \langle 7,2\rangle$

$F+G=0 \therefore G=-F \quad \langle -7,-2\rangle$

Date:

___/___/___

8-4

Find the dot product and the angle between the two vectors if a = 8i-3j and b = 2i-7j.

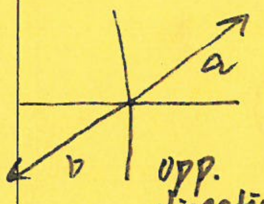
$(8)(2)+(-3)(-7)$
$16+21=37$

$\theta=\cos^{-1}\left(\frac{37}{\|a\|\|b\|}\right)=\frac{37}{(\sqrt{73})(\sqrt{53})}=53^{\circ}30'$

Show that the vectors are orthogonal. **8i – 4j, -6i – 12j**

$(8i-4j)(-6i-12j)=-48+48=0 \Rightarrow$ orthogonal

Show that the vectors are parallel and determine whether they have the same direction or opposite directions. a = <6,18>, b = <-4,-12>



opp. direction

$\cos\theta=\frac{a\cdot b}{\|a\|\|b\|}=\frac{6(-4)+18(-12)}{\sqrt{36+324}\sqrt{16+144}}=\frac{-240}{\sqrt{57,600}}=-1$

$\theta=\cos^{-1}(-1)=\pi$

Determine m such that the two vectors are orthogonal. **5mi + 3j, 2i + 7j**

$(5mi+3j)(2i+7j)=0$

$10m+21=0$

$m=-\frac{21}{10}$

Date:

___/___/___

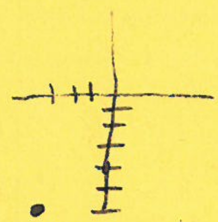
8-5-1

Find the absolute value of |1 – i|. $\sqrt{1^2+(-1)^2}=\sqrt{2}$

Represent the complex number (-3i)(2-i) geometrically.

$-6i+3i^2$

$-6i-3=-3-6i$



Express the complex number in ~~geometric~~ trigono. form with 0≤θ<2π.

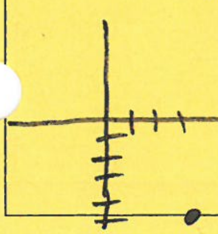
a) 3 - 3√3 i

$z=3-3\sqrt{3}i$

$r=\sqrt{3^2+(-3\sqrt{3})^2}=\sqrt{36}=6$

$\tan\theta=\frac{-3\sqrt{3}}{3}=-\sqrt{3}$

QIV $=\theta=\frac{5\pi}{3}$



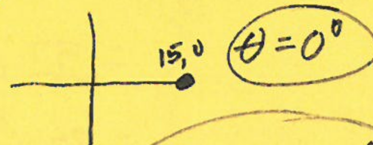
QIV

$z=6\operatorname{cis}\frac{5\pi}{3}$

b) 15

$z=15$

$r=15$



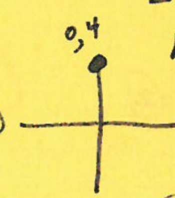
$\theta=0^{\circ}$

$z=15\operatorname{cis}0^{\circ}$

c) 4i

$z=4i$

$r=4$



$\theta=\frac{\pi}{2}$

$z=4\operatorname{cis}\frac{\pi}{2}$

8-5-2

Express the complex number in ~~geometric~~ trigonometric form with $0 \le \theta < 2\pi$.

a) $\sqrt{3} - i$ $\quad z = \sqrt{3} - i$

$$r = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{4} = 2$$

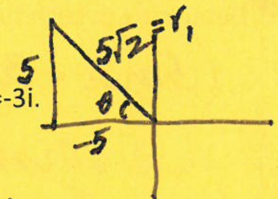
$$\tan\theta = \frac{-1}{\sqrt{3}} ;\ \theta \text{ in QIV} \therefore \theta = \frac{11\pi}{6}$$

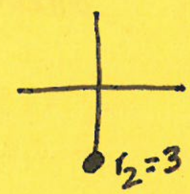
$$z = 2 \operatorname{cis} \frac{11\pi}{6}$$

Express in the form a + bi where a and b are real numbers. $8(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})$.

$$8\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i\right) = 4\sqrt{2} - 4\sqrt{2}\, i$$

Use trig forms to find z_1z_2 and z_1/z_2. $\quad z_1=-5 + 5i$ and $z_2=-3i$.





$$z_1 = 5\sqrt{2} \operatorname{cis} \frac{3\pi}{4} \qquad z_2 = 3 \operatorname{cis} \frac{3\pi}{2}$$

$$z_1 \cdot z_2 = 5\sqrt{2} \cdot 3 \operatorname{cis}\left(\frac{3\pi}{4} + \frac{3\pi}{2}\right)$$

$$= 15\sqrt{2} \operatorname{cis} \frac{9\pi}{4} = 15\sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = 15 + 15i$$

$$\frac{z_1}{z_2} = \frac{5\sqrt{2}}{3} \operatorname{cis}\left(\frac{3\pi}{4} - \frac{3\pi}{2}\right) = \frac{5\sqrt{2}}{3} \operatorname{cis}\left(\frac{-3\pi}{4}\right) = \frac{-5}{3} - \frac{5}{3} i$$

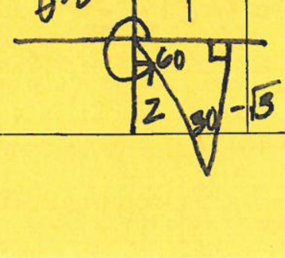
8-6

Use DeMoivre's theorem to change the given complex number to the form a+bi, where a and b are real numbers.

$(1-\sqrt{3}\, i)^5$

$$r = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$\theta = 300 = \frac{5\pi}{3}$$



$$\left(2 \operatorname{cis} \frac{5\pi}{3}\right)^5 = 2^5 \operatorname{cis} \frac{25\pi}{3}$$

$$= 32 \operatorname{cis} \frac{\pi}{3}$$

$$= 32\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$$

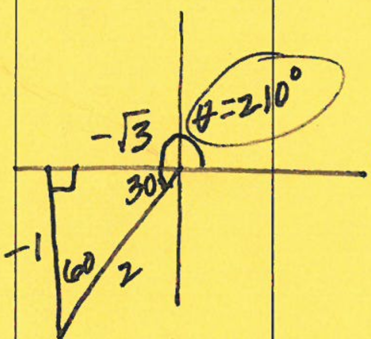
$$= 16 + 16\sqrt{3}\, i$$

8-6 (continued)

Find the five fifth roots of -√3 – i.

$r = \sqrt{(-\sqrt{3})^2 + (-1)^2}$

$= \sqrt{3+1} = \sqrt{4} = 2$



$-\sqrt{3} - i = 2 \text{ cis } 210°$

$w_k = \sqrt[5]{2} \text{ cis} \left(\frac{210 + 360k}{5}\right)$ for $k = 0, 1, 2, 3, 4$

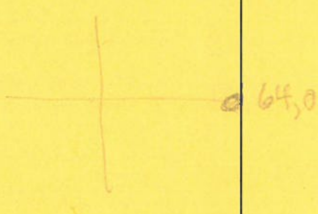
$W_k = \sqrt[5]{2} \text{ cis } \theta$ with $\theta = 42°, 114°, 186°, 258°, 330°$.

$a + bi$

Find the solutions of $x^6 - 64 = 0$. $x^6 = 64$ (find 6 sixth roots of 64)

$64 = 64 + 0i = 64 \text{ cis } 0°$

$w_k = \sqrt[6]{64} \text{ cis} \left(\frac{0 + 360k}{6}\right)$ for $k = 0, 1, 2, 3, 4, 5$



$w_0 = 2 \text{ cis } 0 = 2(1 + 0i) = 2$

$w_1 = 2 \text{ cis } 60° = 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) = 1 + \sqrt{3} i$

$w_2 = 2 \text{ cis } 120° = 2\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) = -1 + \sqrt{3} i$

$w_3 = 2 \text{ cis } 180° = 2(-1 + 0i) = -2$

$w_4 = 2 \text{ cis } 240° = 2\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) = -1 - \sqrt{3} i$

$w_5 = 2 \text{ cis } 300° = 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) = 1 - \sqrt{3} i$