

1. Find all solutions of the equation  $(\cos \theta - 1)(\sin \theta + 1) = 0$ .

$$\cos \theta = 1$$

$$\theta = 2\pi n$$

$$\sin \theta = -1$$

$$\theta = \frac{3\pi}{2} + 2\pi n$$

$$\theta = \frac{\pi}{2} + 2\pi n$$

2. Find the solutions of the equation that are in the interval  $[0, 2\pi]$ .

$$\cos(2x - \frac{\pi}{8}) = 0$$

$$\frac{\pi}{2} + \pi n = 2x - \frac{\pi}{8}$$

$$\frac{5\pi}{8} + \pi n = \frac{2x}{2}$$

$$\frac{5\pi}{16} + \frac{\pi}{2}n = x; n=0,1,2,3$$

$$\frac{5\pi}{16}, \frac{13\pi}{16}, \frac{21\pi}{16}, \frac{29\pi}{16}$$

3. Find the solutions of the equation that are in the interval  $[0, 2\pi]$ .

$$2 \cot t - \csc^2 t = 0$$

$$\frac{2 \cot t}{\sin t} = \frac{1}{\sin^2 t}$$

$$2 \cot t \sin^2 t = \sin t$$

$$2 \cot t \sin^2 t - \sin t = 0$$

$$\sin t(2 \cot t \sin t - 1) = 0$$

$$\sin t = 0; 2 \cot t \sin t = 1$$

double-angle formula  
 $\sin 2t = 1$

$$\frac{2t}{2} = \frac{\pi}{2} + 2\pi n$$

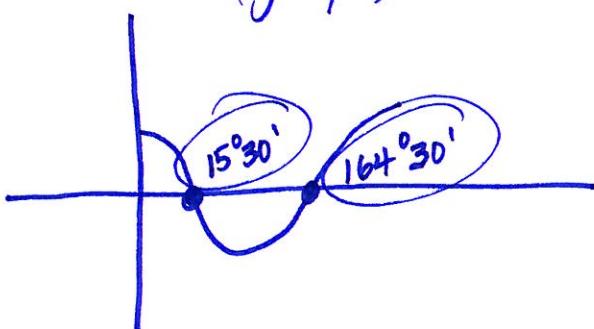
$$t = \frac{\pi}{4} + \pi n; n=0, 1$$

$$t = \frac{\pi}{4}, \frac{5\pi}{4}$$

4. Approximate to the nearest  $10'$  the solutions of the equation in the interval  $[0, 360^\circ]$ .

$$\sin^2 x - 4 \sin x + 1 = 0.$$

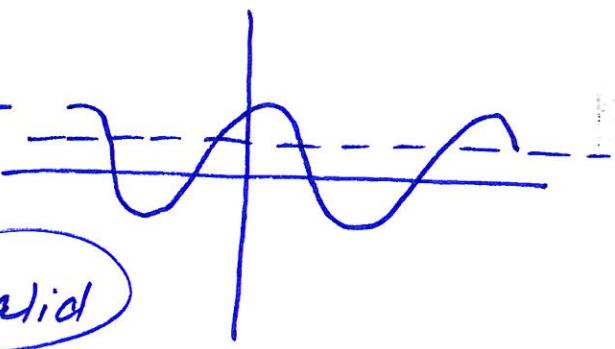
*(graph)*



5. Verify the identity as either valid or invalid.

$$\sqrt{\sin^2 t + \cos^2 t} = \sin t + \cos t$$

-graph-



$$\sqrt{1} \neq \sin t + \cos t$$

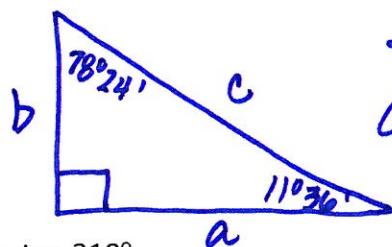
$$1 \neq \sin t + \cos t$$

invalid

6. Express as a cofunction of a complementary angle.

$$\sin 78^\circ 24'$$

$$\cos 11^\circ 36'$$



$$\sin 78^\circ 24' = \frac{a}{c}$$

$$\cos 11^\circ 36' = \frac{a}{c}$$

7. Find the exact value of  $\tan 45^\circ + \tan 210^\circ$ .

$$1 + \frac{1}{\sqrt{3}}$$

$$1 + \frac{\sqrt{3}}{3}$$

8. Find the exact values of  $\sin(\frac{\theta}{2})$ ,  $\cos(\frac{\theta}{2})$ , and  $\tan(\frac{\theta}{2})$  for the given conditions.

$$\sec \theta = -3; 180^\circ < \theta < 270^\circ$$

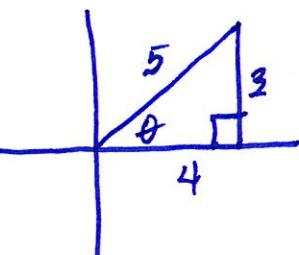
$$\sin \frac{\theta}{2} = +\sqrt{\frac{1-\cos \theta}{2}} = +\sqrt{\frac{1+\frac{1}{3}}{2}} = +\sqrt{\frac{\frac{4}{3}}{2}} = +\sqrt{\frac{4}{6}} = \frac{\pm 2}{\sqrt{6}} =$$

$$\cos \frac{\theta}{2} = -\sqrt{\frac{1+\cos \theta}{2}} = -\sqrt{\frac{1-\frac{1}{3}}{2}} = -\sqrt{\frac{\frac{2}{3}}{2}} = -\sqrt{\frac{2}{6}} = -\sqrt{\frac{1}{3}} = -\frac{\sqrt{3}}{3}$$

$$\tan \frac{\theta}{2} = \frac{1-\cos \theta}{\sin \theta} = \frac{1+\frac{1}{3}}{-\frac{\sqrt{8}}{3}} = \frac{\frac{4}{3}}{-\frac{4\sqrt{8}}{3}} = -\frac{4\sqrt{8}}{8} = -\frac{\sqrt{8}}{2} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$$

9. Find the exact values of  $\sin(\frac{\theta}{2})$ ,  $\cos(\frac{\theta}{2})$ , and  $\tan(\frac{\theta}{2})$  for the given conditions.

$$\sec \theta = \frac{5}{4}; 0^\circ < \theta < 90^\circ. \quad \sin \frac{\theta}{2} = +\sqrt{\frac{1-\cos \theta}{2}} = +\sqrt{\frac{1-\frac{4}{5}}{2}} = \sqrt{\frac{\frac{1}{5}}{2}} = \sqrt{\frac{1}{10}} = \frac{\sqrt{10}}{10}$$



$$\cos \frac{\theta}{2} = +\sqrt{\frac{1+\cos \theta}{2}} = +\sqrt{\frac{1+\frac{4}{5}}{2}} = \sqrt{\frac{\frac{9}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\tan \frac{\theta}{2} = \frac{1-\cos \theta}{\sin \theta} = \frac{1-\frac{4}{5}}{\frac{3}{5}} = \frac{\frac{1}{5}}{\frac{3}{5}} = \frac{1}{3} = \frac{1}{3}$$

10. Find the solutions of the equation that are in the interval  $[0, \frac{2\pi}{5}]$ .

$$\sin 10t + \sin 5t = 0 \quad 2 \sin \frac{10t+5t}{2} \cos \frac{10t-5t}{2} = 0$$

$$2 \sin \frac{15}{2}t \cos \frac{5}{2}t = 0$$

$$\sin \frac{15}{2}t = 0 \quad \cos \frac{5}{2}t = 0$$

$$\left(\frac{2}{15}\right) \frac{15}{2}t = \pi n \left(\frac{2}{15}\right) \quad \left(\frac{2}{5}\right) \frac{5}{2}t = \frac{\pi}{2} + \pi n \left(\frac{2}{5}\right)$$

$$t = \frac{2}{15}\pi n \quad t = \frac{\pi}{5} + \frac{2\pi n}{5}$$

$$n=0, 1, 2$$

$$t = 0, \frac{\pi}{5}, \frac{2\pi}{15}, \frac{4\pi}{15}$$

11. Express as a sum or a difference.

$$\sin 5t \sin 3t \quad \frac{1}{2}(\cos(5t-3t) - \cos(5t+3t))$$

$$\frac{1}{2}(\cos 2t - \cos 8t)$$

12. Express as a product.

$$\sin 3t - \sin 15t \quad 2 \cos \frac{3t+15t}{2} \sin \frac{3t-15t}{2}$$

$$2 \cos 9t \sin(-6t)$$

$$-2 \cos 9t \sin 6t$$

13. Use sum-to-product formulas to find the solutions of the equation  $\cos t = \cos 3t$ .

$$\sin 2t = 0 \quad \sin t = 0 \quad \cos t - \cos 3t = 0$$

$$\frac{2t}{2} = \pi n \quad t = \pi n \quad -2 \sin \frac{t+3t}{2} \sin \frac{t-3t}{2} = 0$$

$$t = \frac{\pi n}{2} \quad \text{this is included in } \frac{\pi n}{2}$$

$$-2 \sin 2t \sin(-t) = 0 \rightarrow \frac{2}{2} \sin 2t \sin t = 0$$

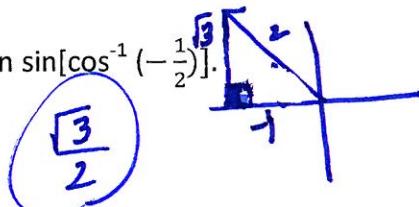
$$(\sin 2t)(\sin t) = 0$$

14. Find the exact values of the expression  $\sin[\arcsin(-\frac{3}{10})]$ .

$$-1 \leq -\frac{3}{10} \leq 1$$

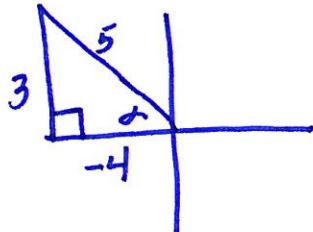
$$\therefore \sin(\sin^{-1}(-\frac{3}{10})) = -\frac{3}{10}$$

15. Find the exact values of the expression  $\sin[\cos^{-1}(-\frac{1}{2})]$ .



### sin 2θ

16. Find the exact values of the expression  $\sin[2 \arccos(-\frac{4}{5})]$ .



$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ &= 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = \frac{-24}{25}\end{aligned}$$

17. Use inverse trig functions to find the solutions of the equation that are in the given interval.  $[0, 2\pi)$

$$(\cos x)(15 \cos x + 4) = 0$$

$$15 \cos^2 x + 4 \cos x - 3 = 0$$

$$(5 \cos x + 3)(3 \cos x - 1) = 0$$

$$\cos x = -\frac{3}{5}$$

$$x = \cos^{-1}\left(-\frac{3}{5}\right)$$

$$x = 2.21$$

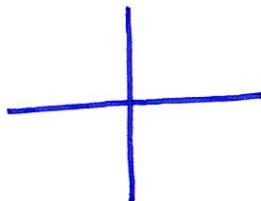
$$2\pi - 2.21 = 4.07$$

$$\cos x = \frac{1}{3}$$

$$x = \cos^{-1}\left(\frac{1}{3}\right)$$

$$x = 1.23$$

$$2\pi - 1.23 = 5.05$$



18. Verify the identity.  $\frac{1 + \sec 4x}{\sin 4x + \tan 4x} = \csc 4x$ .

$$\begin{aligned}\frac{1 + \frac{1}{\cos 4x}}{\sin 4x + \frac{\sin 4x}{\cos 4x}} &= \\ \frac{\frac{\cos 4x + 1}{\cos 4x}}{\frac{\sin 4x \cos 4x + \sin 4x}{\cos 4x}} &= \frac{\cos 4x + 1}{\sin 4x (\cos 4x + 1)} \\ &= \frac{1}{\sin 4x} = \\ \csc 4x &= \csc 4x\end{aligned}$$

19. Verify the identity.  $\ln \sec \theta = -\ln \cos \theta$

$$\ln \sec \theta = -\ln \cos \theta$$

$$\ln \frac{1}{\cos \theta} = -\ln \cos \theta$$

$$\ln(1) - \ln(\cos \theta) = -\ln \cos \theta$$

$$0 - \ln(\cos \theta) = -\ln \cos \theta$$

$$-\ln(\cos \theta) = -\ln \cos \theta$$