***AAT (IC/HW)-Day 1**

Chapter 9: 9-5 Systems of Linear Equations in More Than Two Variables

Name Key

Date: ________ **Period:** ________

Use matrices to solve the system.

1. $\begin{cases} x - 2y - 3z = -1 \\ 2x + y + z = 6 \\ x + 3y - 2z = 13 \end{cases}$

$A = \begin{bmatrix} 1 & -2 & -3 \\ 2 & 1 & 1 \\ 1 & 3 & -2 \end{bmatrix}$ $B = \begin{bmatrix} -1 \\ 6 \\ 13 \end{bmatrix}$

$A^{-1} \cdot B = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

$(2, 3, -1)$

2. $\begin{cases} 5x + 2y - z = -7 \\ x - 2y + 2z = 0 \\ 3y + z = 17 \end{cases}$

$[A]^{-1} \cdot [B] = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$

$(-2, 4, 5)$

3. $\begin{cases} 2x + 6y - 4z = 1 \\ x + 3y - 2z = 4 \\ 2x + y - 3z = -7 \end{cases}$

$[A]^{-1} \cdot [B]$

no solution

4. $\begin{cases} 2x - 3y + 2z = -3 \\ -3x + 2y + z = 1 \\ 4x + y - 3z = 4 \end{cases}$

$[A]^{-1} \cdot [B] = \begin{bmatrix} \frac{2}{3} \\ \frac{31}{21} \\ \frac{1}{21} \end{bmatrix}$

$\left(\frac{2}{3}, \frac{31}{21}, \frac{1}{21}\right)$

5. $\begin{cases} x + 3y + z = 0 \\ x + y - z = 0 \\ x - 2y - 4z = 0 \end{cases}$ – use elimination

① $x + 3y + z = 0$
$- \quad 3x + 3y - 3z = 0$
$-2x + 4z = 0$
$x - 2z = 0 \to x = 2z \to x = 2c$

② $\cancel{x} + 3y + z = 0$
$- \quad \cancel{x} + y - z = 0$
$2y + 2z = 0$
$y + z = 0 \to y = -z \to y = -c$

$z = c$

$(2c, -c, c)$ for all R's

6. $\begin{cases} 2x + y + z = 0 \\ x - 2y - 2z = 0 \\ x + y + z = 0 \end{cases}$ – use elimination

① $\cancel{x} - 2y - 2z = 0$
$- \quad \cancel{x} + y + z = 0$
$-3y - 3z = 0$
$y + z = 0 \to y = -z$
$y = c$

② $2x + \cancel{y} + \cancel{z} = 0$
$- \quad x + \cancel{y} + \cancel{z} = 0$
$x = 0$

$z = c$

$(0, -c, c)$

7. $\begin{cases} 3x - 2y + 5z = 7 \\ x + 4y - z = -2 \end{cases}$

$3x - 2y + 5z = 7 \to \cancel{3x} - 2y + 5z = 7$
$3(x + 4y - z = -2) \to \cancel{3x} + 12y - 3z = -6$
$-14y + 8z = 13$
$y = \frac{4}{7}z - \frac{13}{14}$

$z = c$

$6x - \cancel{4y} + 10z = 14$
$+ \quad x + \cancel{4y} - z = -2$
$7x + 9z = 12$
$x = -\frac{9}{7}z + \frac{12}{7}$

$\left(-\frac{9}{7}c + \frac{12}{7}, \frac{4}{7}c - \frac{13}{14}, c\right)$

8. $\begin{cases} 5x + 2z = 1 \\ y - 3z = 2 \\ 2x + y = 3 \end{cases}$

$[A]^{-1} \cdot [B] = \begin{bmatrix} \frac{1}{11} \\ \frac{31}{11} \\ \frac{3}{11} \end{bmatrix}$

$\left(\frac{1}{11}, \frac{31}{11}, \frac{3}{11}\right)$

9. $\begin{cases} 2x + 3y = 5 \\ x - 3y = 4 \\ x + y = -2 \end{cases}$

No Solution

10. Three solutions contain a certain acid. The first contains 10% acid, the second 30%, and the third 50%. A chemist wishes to use all three solutions to obtain a 50-liter mixture containing 32% acid. If the chemist wants to use twice as much of the 50% solution as of the 30% solution, how many liters of each solution should be used?

$x+y+z=50$

$.10x+.30y+.50z=(.32)(50)$

$z=2y \rightarrow -2y+z=0$

$$\begin{bmatrix} 1 & 1 & 1 & 50 \\ .10 & .30 & .50 & 16 \\ 0 & -2 & 1 & 0 \end{bmatrix}$$

$x=17$ liters

$y=11$ liters

$z=22$ liters

11. A swimming pool can be filled by three pipes, A, B, and C. Pipe A alone can fill the pool in 8 hours. If pipes A and C are used together, the pool can be filled in 6 hours; if B and C are used together, it takes 10 hours. How long does it take to fill the pool if all three pipes are used?

z = # hours for pipe C to fill pool alone

y = # hours for pipe B to fill pool alone

$\frac{1}{8}+\frac{1}{z}=\frac{1}{6}$; $\frac{1}{z}=\frac{4}{24}-\frac{3}{24}$

$\frac{1}{z}=\frac{1}{24}$

$z=24$

$\frac{1}{y}+\frac{1}{24}=\frac{1}{10}$

$\frac{1}{y}=\frac{12}{120}-\frac{5}{120}$

$\frac{1}{y}=\frac{7}{120}$; $y=\frac{120}{7}$

x = # hours for all three pipes to fill pool

$\frac{1}{8}+\frac{7}{120}+\frac{1}{24}=\frac{1}{x}$

$\frac{1}{x}=\frac{15}{120}+\frac{7}{120}+\frac{5}{120}$

$x=120/27=40/9$

12. A shop specializes in preparing blends of gourmet coffees. From Colombian, Brazilian, and Kenyan coffees, the owner wishes to prepare 1-pound bags that will sell for \$8.50. The cost per pound of these coffees is \$10, \$6, and \$8, respectively. The amount of Columbian is to be three times the amount of Brazilian. Find the amount of each type of coffee in the blend.

$x, y, z \rightarrow$ amount of Columbian, Brazilian, Kenyan coffee

$x+y+z=1$

$10x+6y+8z=(8.5)(1)$

$x=3y \rightarrow x-3y=0$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 10 & 6 & 8 & 8.5 \\ 1 & -3 & 0 & 0 \end{bmatrix}$$

$x=\frac{3}{8}$

$y=\frac{1}{8}$

$z=\frac{1}{2}$