

Use substitution to solve the system.

1. $\begin{cases} y = x^2 - 4 \\ y = 2x - 1 \end{cases}$

$$2x - 1 = x^2 - 4$$

$$0 = x^2 - 2x - 3$$

$$0 = (x - 3)(x + 1) = 0$$

$$\begin{matrix} x = 3 & x = -1 \\ y = 5 & y = -3 \end{matrix}$$

$(3, 5)$ $(-1, -3)$

2. $\begin{cases} y^2 = 1 - x \\ x + 2y = 1 \end{cases} \rightarrow y = 1 - 2y$

$$y^2 = 1 - (1 - 2y)$$

$$y^2 = 1 - 1 + 2y$$

$$y^2 - 2y = 0$$

$$y(y - 2) = 0$$

$$\begin{matrix} y = 0 & y = 2 \\ x = 1 & x = -3 \end{matrix}$$

$(1, 0)$ $(-3, 2)$

3. $\begin{cases} 2y = x^2 \\ y = 4x^3 \end{cases}$

$$2(4x^3) = x^2$$

$$8x^3 = x^2$$

$$8x^3 - x^2 = 0$$

$$x^2(8x - 1) = 0$$

$$\begin{matrix} x = 0 & 8x - 1 = 0 \\ y = 0 & 8x = 1 \\ & x = \frac{1}{8} \\ & y = \frac{1}{128} \end{matrix}$$

$(0, 0)$ $(\frac{1}{8}, \frac{1}{128})$

4. $\begin{cases} x + 2y = -1 \\ 2x - 3y = 12 \end{cases} \rightarrow x = -2y - 1$

$$2(-2y - 1) - 3y = 12$$

$$-4y - 2 - 3y = 12$$

$$-7y = 14$$

$$y = -2$$

$$x = 3$$

$(3, -2)$

5. $\begin{cases} 2x - 3y = 1 \\ -6x + 9y = 4 \end{cases} \rightarrow \begin{matrix} -3y = -2x + 1 \\ -3 & -3 \end{matrix}$
 $y = \frac{2}{3}x - \frac{1}{3}$

$$-6x + 9(\frac{2}{3}x - \frac{1}{3}) = 4$$

$$-6x + 6x - 3 = 4$$

$$-3 \neq 4$$

\emptyset

6. $\begin{cases} x + 3y = 5 \\ x^2 + y^2 = 25 \end{cases} \rightarrow x = 5 - 3y$

$$(5 - 3y)^2 + y^2 = 25$$

$$25 - 30y + 9y^2 + y^2 = 25$$

$$10y^2 - 30y = 0$$

$$10y(y - 3) = 0$$

$$\begin{matrix} 10y = 0 & y - 3 = 0 \\ y = 0 & y = 3 \\ x = 5 & x = -4 \end{matrix}$$

$(5, 0)$ $(-4, 3)$

$$7. \begin{cases} x^2 + y^2 = 8 \\ y - x = 4 \end{cases} \rightarrow y = x + 4$$

$$\begin{aligned} x^2 + (x+4)^2 &= 8 \\ x^2 + x^2 + 8x + 16 &= 8 \\ 2x^2 + 8x + 8 &= 0 \\ 2(x^2 + 4x + 4) &= 0 \\ 2(x+2)^2 &= 0 \\ (x+2) &= 0 \\ x &= -2 \\ y &= 2 \end{aligned}$$

$(-2, 2)$

$$8. \begin{cases} x^2 + y^2 = 9 \\ y - 3x = 2 \end{cases} \rightarrow y = 3x + 2$$

$$\begin{aligned} x^2 + (3x+2)^2 &= 9 \\ x^2 + 9x^2 + 12x + 4 &= 9 \\ 10x^2 + 12x - 5 &= 0 \end{aligned}$$

Use quadratic program

$$\begin{aligned} x &\approx .327 & x &\approx -1.527 \\ y &\approx 2.98 & y &\approx -2.58 \end{aligned}$$

$(.327, 2.98)$ $(-1.527, -2.58)$

$$9. \begin{cases} x^2 + y^2 = 16 \\ 2y - x = 4 \end{cases} \rightarrow 2y - 4 = x$$

$$\begin{aligned} (2y-4)^2 + y^2 &= 16 \\ 4y^2 - 16y + 16 + y^2 &= 16 \\ 5y^2 - 16y &= 0 \\ y(5y-16) &= 0 \end{aligned}$$

$$\begin{aligned} y &= 0 & 5y - 16 &= 0 \\ x &= -4 & y &= \frac{16}{5} \\ & & x &= \frac{12}{5} \end{aligned}$$

$(-4, 0)$ $(\frac{12}{5}, \frac{16}{5})$

$$10. \begin{cases} (x-1)^2 + (y+2)^2 = 10 \\ x + y = 1 \end{cases} \rightarrow x = 1 - y$$

$$\begin{aligned} (1-y-1)^2 + (y+2)^2 &= 10 \\ y^2 + y^2 + 4y + 4 &= 10 \\ 2y^2 + 4y - 6 &= 0 \\ 2(y^2 + 2y - 3) &= 0 \\ 2(y+3)(y-1) &= 0 \\ y+3=0 & \quad y-1=0 \\ y=-3 & \quad y=1 \\ x=4 & \quad x=0 \end{aligned}$$

$(4, -3)$ $(0, 1)$

$$11. \begin{cases} y = 20/x^2 \\ y = 9 - x^2 \end{cases}$$

$$\begin{aligned} \frac{20}{x^2} &= \frac{9-x^2}{1} \\ 20 &= 9x^2 - x^4 \\ x^4 - 9x^2 + 20 &= 0 \\ (x^2-4)(x^2-5) &= 0 \\ x^2-4=0 & \quad x^2-5=0 \\ x^2=4 & \quad x^2=5 \\ x=\pm 2 & \quad x=\pm\sqrt{5} \\ y=5 & \quad y=4 \end{aligned}$$

$(\pm 2, 5)$ $(\pm\sqrt{5}, 4)$

$$12. \begin{cases} x^2 - y^2 = 4 \\ x^2 + y^2 = 12 \end{cases} \rightarrow x^2 = y^2 + 4$$

$$\begin{aligned} y^2 + 4 + y^2 &= 12 \\ \frac{2y^2}{2} &= \frac{8}{2} \\ y^2 &= 4 \\ y &= \pm 2 \\ x &= \pm 2\sqrt{2} \end{aligned}$$

$(2\sqrt{2}, \pm 2)$ $(-2\sqrt{2}, \pm 2)$

13. A rancher has 2420 feet of fence to enclose a rectangular region that lies along a straight river. If no fence is used along the river, is it possible to enclose 10 acres of land? Recall that 1 acre = 43,560 ft².



$$\underline{10 \text{ acres} = 435,600 \text{ ft.}^2}$$

$$(2420 - 2w)w = 435,600$$

$$2w^2 - 2420w + 435,600 = 0$$

$$2(w^2 - 1210w + 217,800) = 0$$

$$(w - 990)(w - 220) = 0$$

$$\begin{matrix} w = 990 & \text{or} & w = 220 \\ l = 440 & & l = 1980 \end{matrix}$$

$$l + 2w = 2420 \rightarrow l = 2420 - 2w$$

$$lw = 435,600$$

14. Solve the system of equations (a) graphically and (b) using substitution. Compare your answers.

$$\begin{matrix} x^2 + y^2 = 4 & x + y = 1 \\ y = \pm\sqrt{4 - x^2} & y = 1 - x \end{matrix}$$

$$\begin{cases} x^2 + y^2 = 4 \\ x + y = 1 \end{cases} \rightarrow y = 1 - x$$

$$(b) x^2 + (1 - x)^2 = 4$$

$$x^2 + 1 - 2x + x^2 = 4$$

$$2x^2 - 2x - 3 = 0$$

(quadratic formula)

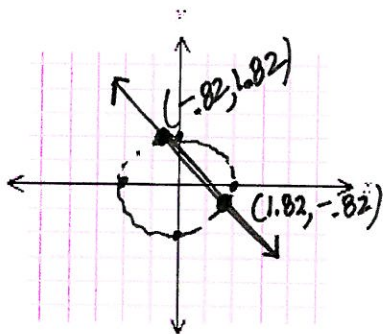
$$= \frac{2 \pm \sqrt{4 - 4(2)(-3)}}{2(2)}$$

$$= \frac{2 \pm \sqrt{28}}{4} = \frac{2 \pm 2\sqrt{7}}{4}$$

$$x = \frac{1 \pm \sqrt{7}}{2} \approx -0.82, 1.82$$

$$y = 1.82, -0.82$$

$$\begin{matrix} (-0.82, 1.82) & (1.82, -0.82) \end{matrix}$$



(a)