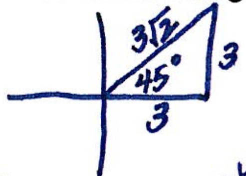
***AAT (IC/HW)-Days 1-2** Name: Key

Chapter 8: 8-6 De Moivre's Theorem & nth Roots of Complex #'s Date: ____________ Period: ____________

Use De Moivre's theorem to change the given complex number to the form a + bi, where a and b are real numbers.

1. $(3 + 3i)^5$



$r = 3\sqrt{2}$

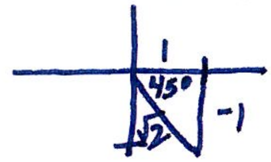
$= (3\sqrt{2}\text{ cis } \frac{\pi}{4})^5$

$= (3\sqrt{2})^5 \text{ cis } \frac{5\pi}{4}$

$= 972\sqrt{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$

$= -972 - 972i$

2. $(1 - i)^{10}$



$r = \sqrt{2}$

$= (\sqrt{2}\text{ cis } \frac{7\pi}{4})^{10}$

$= (\sqrt{2})^{10} \text{ cis } \frac{35\pi}{2}$

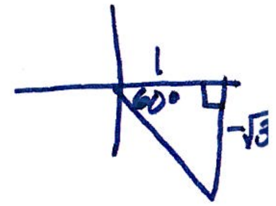
$= 32 \text{ cis } \frac{3\pi}{2}$

$= 32(0 - i)$

$= 0 - 32i$

$= -32i$

3. $(1 - \sqrt{3}\, i)^3$



$r = 2$

$= (2 \text{ cis } \frac{5\pi}{3})^3$

$= 2^3 \text{ cis } 5\pi$

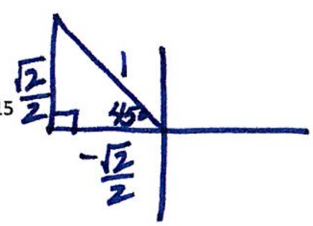
$= 8 \text{ cis } \pi$

$= 8(-1 + 0i)$

$= -8 + 0i$

$= -8$

4. $\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^{15}$



$r = 1$

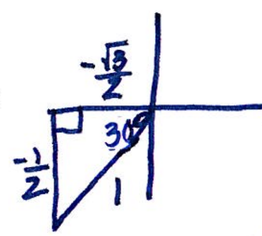
$= (1 \text{ cis } \frac{3\pi}{4})^{15}$

$= 1^{15} \text{ cis } \frac{45\pi}{4}$

$= 1 \text{ cis } \frac{5\pi}{4}$

$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

5. $\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^{20}$



$r = 1$

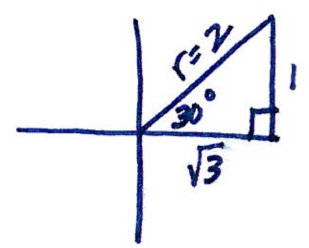
$= (1 \text{ cis } \frac{7\pi}{6})^{20}$

$= 1^{20} \text{ cis } \frac{70\pi}{3}$

$= 1 \text{ cis } \frac{4\pi}{3}$

$= -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

6. $(\sqrt{3} + i)^7$



$r = 2$

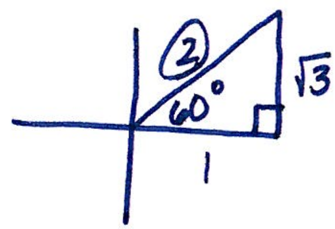
$= (2 \text{ cis } \frac{\pi}{6})^7$

$= 2^7 \text{ cis } \frac{7\pi}{6}$

$= 128\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$

$= -64\sqrt{3} - 64i$

7. Find the two square roots of $1 + \sqrt{3}\,i$.



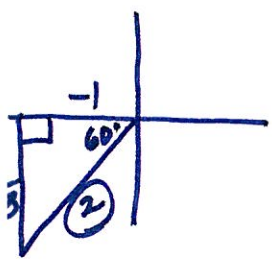
$$= 2 \text{ cis } 60^\circ$$

$$W_k = \sqrt{2} \text{ cis}\left(\frac{60^\circ + 360^\circ k}{2}\right) \quad \text{for } k = 0, 1$$

$$W_0 = \sqrt{2} \text{ cis } 30^\circ = \sqrt{2}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$$

$$W_1 = \sqrt{2} \text{ cis } 210^\circ = \sqrt{2}\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = -\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i$$

8. Find the four fourth roots of $-1 - \sqrt{3}\,i$.



$$= 2 \text{ cis } 240^\circ$$

$$W_k = \sqrt[4]{2} \text{ cis}\left(\frac{240^\circ + 360^\circ k}{4}\right) \quad \text{for } k = 0, 1, 2, 3$$

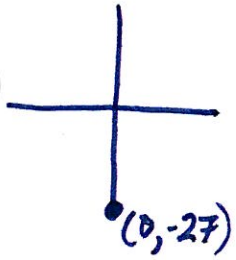
$$W_0 = \sqrt[4]{2} \text{ cis } 60^\circ = \sqrt[4]{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

$$W_1 = \sqrt[4]{2} \text{ cis } 150^\circ = \sqrt[4]{2}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$$

$$W_2 = \sqrt[4]{2} \text{ cis } 240^\circ = \sqrt[4]{2}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

$$W_3 = \sqrt[4]{2} \text{ cis } 330^\circ = \sqrt[4]{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$$

9. Find the three cube roots of -27i.



$= 27 \text{ cis } 270°$

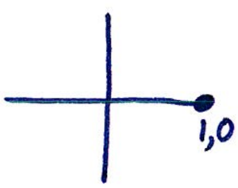
$W_K = \sqrt[3]{27} \text{ cis}\left(\frac{270° + 360K}{3}\right)$ for $K = 0, 1, 2$

$W_0 = 3 \text{ cis } 90° = 3(0 + i) = 3i$

$W_1 = 3 \text{ cis } 210° = 3\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \frac{-3\sqrt{3}}{2} - \frac{3}{2}i$

$W_2 = 3 \text{ cis } 330° = 3\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \frac{3\sqrt{3}}{2} - \frac{3}{2}i$

10. Find the six sixth roots of unity. = 1



$= 1 \text{ cis } 0°$

$W_K = \sqrt[6]{1} \text{ cis}\left(\frac{0° + 360K}{6}\right)$ for $K = 0, 1, 2, 3, 4, 5$

$W_0 = 1 \text{ cis } 0° = 1 + 0i$

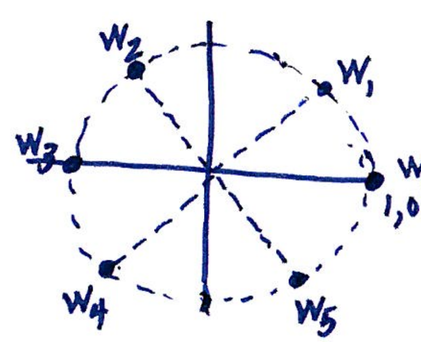
$W_1 = 1 \text{ cis } 60° = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$W_2 = 1 \text{ cis } 120° = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

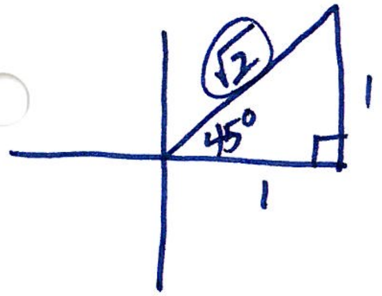
$W_3 = 1 \text{ cis } 180° = -1 + 0i$

$W_4 = 1 \text{ cis } 240° = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

$W_5 = 1 \text{ cis } 300° = \frac{1}{2} - \frac{\sqrt{3}}{2}i$



11. Find the five fifth roots of 1 + i.



$r = \sqrt{2}$

$\sqrt{2} \operatorname{cis} 45°$

$W_K = \sqrt{\sqrt[5]{2}} \operatorname{cis}\left(\frac{45 + 360K}{5}\right)$ for $K = 0, 1, 2, 3, 4$

$W_K = \sqrt[10]{2} \operatorname{cis} \theta$ with $\theta = 9°, 81°, 153°, 225°, 297°$ *

* May leave answer like this since not exact values!

Find the solutions of the equation.

12. $x^4 - 16 = 0$

Algebraically

$(x^2 - 4)(x^2 + 4) = 0$

$x^2 - 4 = 0$ $\quad$ $x^2 + 4 = 0$

$x^2 = 4$ $\quad$ $x^2 = -4$

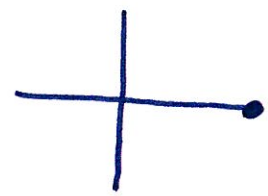
$x = \pm 2$ $\quad$ $x = \pm 2i$

-or-

Find 4 fourth roots of 16.

nth roots

$16 + 0i$



$r = 16$

$16 \operatorname{cis} 0°$

$W_K = \sqrt[4]{16} \operatorname{cis}\left(\frac{0° + 360K}{4}\right)$ for $K = 0, 1, 2, 3$

$W_0 = 2 \operatorname{cis} 0 = 2(1 + 0i) = 2$

$W_1 = 2 \operatorname{cis} 90° = 2(0 + i) = 2i$

$W_2 = 2 \operatorname{cis} 180° = 2(-1 + 0i) = -2$

$W_3 = 2 \operatorname{cis} 270° = 2(0 - i) = -2i$