*AAT (IC/HW)-Days 1-2

Chapter 8: 8-5 Trigonometric Form for Complex Numbers

Name Key

Date: _____ Period: _____

Find the absolute value.

1. $|3 - 4i|$

$\sqrt{9+16}$

$= \sqrt{25} = 5$

2. $|-6 - 7i|$

$\sqrt{36+49}$

$= \sqrt{85}$

3. $|8i|$

$\sqrt{0+64}$

$= \sqrt{64} = 8$

4. $|i^{500}|$

$= (i^4)^{125}$

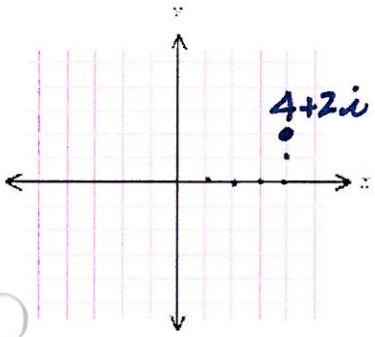
$= (1)^{125}$

$= |1+0i|$

$= \sqrt{1^2+0^2} = \sqrt{1} = 1$

Represent the complex number geometrically.

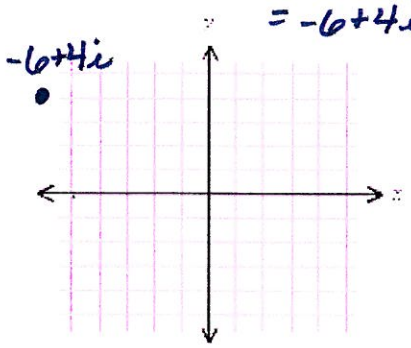
5. $4+2i$



6. $2i(2 + 3i)$

$4i + 6i^2 = 4i - 6$

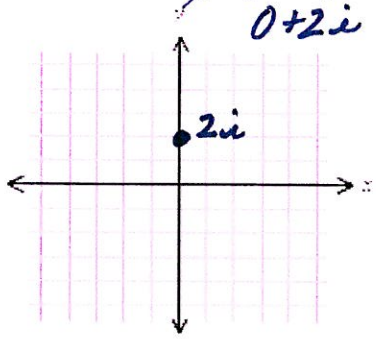
$= -6 + 4i$



7. $(1 + i)^2 = 1 + 2i + i^2$

$1 + 2i - 1$

$0 + 2i$



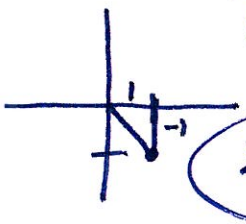
Express the complex number in trigonometric form with $0 \le \theta \le 2\pi$.

8. $1 - i$

$r = \sqrt{1+(-1)^2} = \sqrt{2}$

$\tan\theta = \frac{-1}{1} = -1$

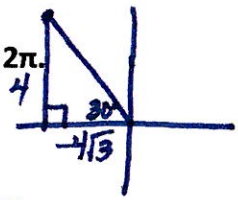
$\theta = -\frac{\pi}{4}$ or $\frac{7\pi}{4}$



$z = \sqrt{2}\,\text{cis}\,\frac{7\pi}{4}$

QIV

9. $-4\sqrt{3} + 4i$



$r = \sqrt{16 \cdot 3 + 16}$

$= \sqrt{64} = 8$

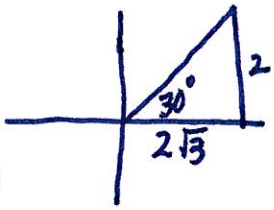
$\tan\theta = \frac{4}{-4\sqrt{3}} = \frac{-1}{\sqrt{3}}$

$\theta = \frac{5\pi}{6}$

$z = 8\,\text{cis}\,\frac{5\pi}{6}$

QII

10. $2\sqrt{3} + 2i$



$r = \sqrt{4 \cdot 3 + 4}$

$= \sqrt{16} = 4$

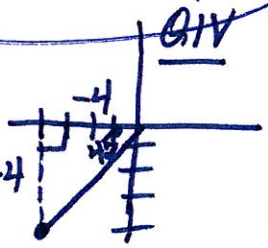
$\tan\theta = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$

$z = 4\,\text{cis}\,\frac{\pi}{6}$

QI

11. $-4 - 4i$



$= \sqrt{(-4)^2 + (-4)^2}$

$r = \sqrt{32}$

$r = 4\sqrt{2}$

$\tan\theta = \frac{-4}{-4} = 1$

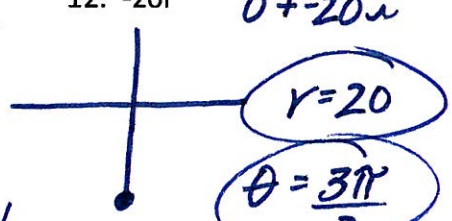
$\theta = \frac{5\pi}{4}$

$z = 4\sqrt{2}\,\text{cis}\,\frac{5\pi}{4}$

QIII

12. $-20i$

$0 + -20i$



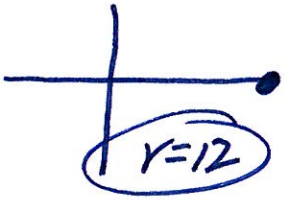
$r = 20$

$\theta = \frac{3\pi}{2}$

$z = 20\,\text{cis}\,\frac{3\pi}{2}$

13. 12

$12 + 0i$



$r = 12$

$\theta = 0$

$z = 12\,\text{cis}\,0$

Express in the form a + bi, where a and b are real numbers.

14. $4(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$

$= 4\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$

$= 2\sqrt{2} + 2\sqrt{2}i$

15. $6(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$

$6\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

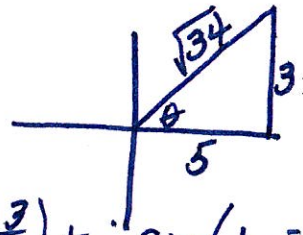
$= -3 + 3\sqrt{3}i$

16. $5(\cos\pi + i\sin\pi)$

$5(-1 + 0i)$

$= -5$

17. $\sqrt{34}\,\text{cis}\,(\tan^{-1}\frac{3}{5})$

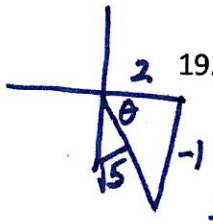


$= \sqrt{34}\left[\cos(\tan^{-1}\frac{3}{5}) + i\sin(\tan^{-1}\frac{3}{5})\right]$

$= \sqrt{34}\left(\frac{5}{\sqrt{34}} + \frac{3}{\sqrt{34}}i\right)$

$= 5 + 3i$

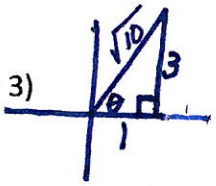
18. $\sqrt{5}\,\text{cis}\,[\tan^{-1}(-\frac{1}{2})]$



$= \sqrt{5}\left(\frac{2}{\sqrt{5}} + \frac{-1}{\sqrt{5}}i\right)$

$= 2 - i$

19. $\sqrt{10}\,\text{cis}\,(\tan^{-1}3)$

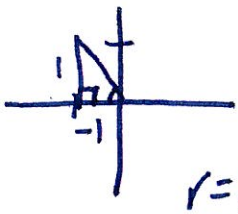


$= \sqrt{10}\left(\frac{1}{\sqrt{10}} + \frac{3}{\sqrt{10}}i\right)$

$= 1 + 3i$

Use trigonometric forms to find z_1z_2 and z_1/z_2.

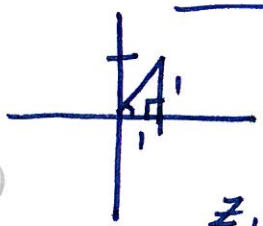
20. $z_1 = -1 + i$, $z_2 = 1 + i$



$z_1 = -1 + i = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$z_2 = 1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$r = \sqrt{2}$



$z_1 \cdot z_2 = (\sqrt{2})(\sqrt{2})\left(\cos\left(\frac{3\pi}{4} + \frac{\pi}{4}\right) + i\sin\left(\frac{3\pi}{4} + \frac{\pi}{4}\right)\right)$

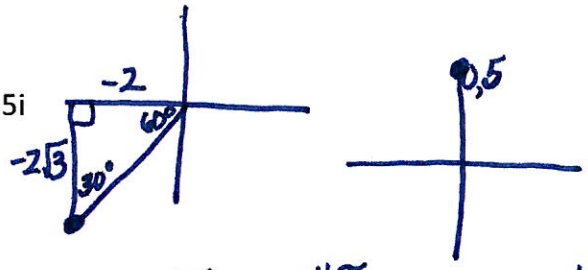
$= 2(\cos\pi + i\sin\pi)$

$= 2(-1 + 0i) = -2 + 0i$

$\frac{z_1}{z_2} = \frac{\sqrt{2}}{\sqrt{2}}\left(\cos\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) + i\sin\left(\frac{3\pi}{4} - \frac{\pi}{4}\right)\right)$

$= 1\,\text{cis}\,\frac{\pi}{2} = 0 + i$

21. $z_1 = -2 - 2\sqrt{3}\,i$, $z_2 = 5i$



$r = \sqrt{4+4\cdot 3}$

$r = \sqrt{16} = 4$

$z_1 = 4\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

$z_2 = 5\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$z_1 z_2 = 4\cdot 5\left(\cos\left(\frac{4\pi}{3}+\frac{\pi}{2}\right) + i\sin\left(\frac{4\pi}{3}+\frac{\pi}{2}\right)\right)$

$= 20\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$

$= 10\sqrt{3} - 10i$

$\frac{z_1}{z_2} = \frac{4}{5}\left(\cos\left(\frac{4\pi}{3}-\frac{\pi}{2}\right) + i\sin\left(\frac{4\pi}{3}-\frac{\pi}{2}\right)\right)$

$= \frac{4}{5}\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

$= \frac{-2\sqrt{3}}{5} + \frac{2}{5}i$

The trigonometric form of complex numbers is often used by electrical engineers to describe the current I, voltage V, and impedance Z in electrical circuits with alternating current. Impedance is the opposition to the flow of current in a circuit. The relationship among these three quantities is **I = V/Z**. Approximate the unknown quantity, and express the answer in rectangular form to two decimal places.

22. (Finding voltage) I = 10 cis 35°, Z = 3 cis 20°

unknown

$I = \frac{V}{Z}$

$V = IZ$

$= (10\,\text{cis}\,35^\circ)\cdot(3\,\text{cis}\,20^\circ)$

$= (10\cdot 3)\,\text{cis}\,(35^\circ + 20^\circ)$

$= 30\,\text{cis}\,55^\circ \approx 17.21 + 24.57i$

23. (Finding impedance) I = 8 cis 5°, V = 115 cis 45°

unknown

$I = \frac{V}{Z}$

$Z = \frac{V}{I} = \frac{115\,\text{cis}\,45^\circ}{8\,\text{cis}\,5^\circ} = (115\div 8)\,\text{cis}\,(45^\circ - 5^\circ)$

$= 14.375\,\text{cis}\,40^\circ$

$\approx 11.01 + 9.24i$