*AAT (IC/HW)-Days 1-2
Chapter 8: 8-4 The Dot Product

Name Key
Date: _____ Period: _____

Find (a) the dot product of the two vectors and (b) the angle between the two vectors.

1. <-2, 5>, <3, 6>

(a) $(-2)(3)+(5)(6)$
$= -6+30$
$= 24$

(b) $\|a\| = \sqrt{4+25} = \sqrt{29}$
$\|b\| = \sqrt{9+36} = \sqrt{45}$
$\theta = \cos^{-1}\left(\frac{24}{\sqrt{29}\cdot\sqrt{45}}\right)$
$\theta \approx 48.36^\circ$

2. 4i - j, -3i + 2j

(a) $(4)(-3)+(-1)(2)$
$= -12-2$
$= -14$

(b) $\theta = \cos^{-1}\left(\frac{-14}{\sqrt{17}\cdot\sqrt{13}}\right)$
$\theta \approx 160.35^\circ$

3. 9i, 5i + 4j

(a) $(9)(5)+0(4)$
$= 45+0$
$= 45$

(b) $\theta = \cos^{-1}\left(\frac{45}{\sqrt{81}\cdot\sqrt{41}}\right)$
$\theta \approx 38^\circ 40'$

4. <10, 7>, $<-2, -\frac{7}{5}>$

(a) $(10)(-2)+(7)\left(-\frac{7}{5}\right)$
$= -20 + -\frac{49}{5}$
$= \frac{-149}{5} = -29.8$

(b) $\theta = \cos^{-1}\left(\frac{-149/5}{\sqrt{149}\cdot\sqrt{149/25}}\right)$
$\theta \approx \cos^{-1}(-1)$
$\theta = 180^\circ$

Show that the vectors are orthogonal.

5. <4, -1>, <2, 8>

$(4)(2)+(-1)(8)$
$= 8 + -8$
$= 0 \Rightarrow$ vectors are orthogonal

6. -4j, -7i

$(0)(-7)+(0)(-4)$
$= 0+0$
$= 0 \Rightarrow$ vectors are orthogonal!

Show that the vectors are parallel, and determine whether they have the same direction or opposite directions.

7. $\mathbf{a} = 3i - 5j$, $\mathbf{b} = -\frac{12}{7}i + \frac{20}{7}j$

$\cos\theta = \frac{a\cdot b}{\|a\|\,\|b\|} = \frac{(3)\left(\frac{-12}{7}\right)+(-5)\left(\frac{20}{7}\right)}{\sqrt{9+25}\cdot\sqrt{\frac{144}{49}+\frac{400}{49}}}$

$= \frac{-\frac{136}{7}}{\frac{136}{7}} = -1$ (if sketch... (opp. direction))

$\theta = \cos^{-1}(-1) = \pi \Rightarrow$ vectors are $\parallel$

8. $\mathbf{a} = \langle\frac{2}{3}, \frac{1}{2}\rangle$, $\mathbf{b} = \langle 8, 6\rangle$

$\cos\theta = \frac{(\frac{2}{3})(8)+(\frac{1}{2})(6)}{\sqrt{\frac{4}{9}+\frac{1}{4}}\cdot\sqrt{64+36}} = \frac{\frac{25}{3}}{\frac{25}{3}} = 1$

$\theta = \cos^{-1}(1) = 0^\circ \Rightarrow$ vectors are $\parallel$.

(if sketch... (same direction))

Determine m such that the two vectors are orthogonal.

9. $3i - 2j$, $4i + 5mj$

* Need Dot Product = 0

$12 - 10m = 0$

$\frac{12}{10} = \frac{10m}{10}$

$\frac{6}{5} = m$

10. $9i - 16mj$, $i + 4mj$

$9 - 64m^2 = 0$

$\frac{-64m^2}{-64} = \frac{-9}{-64}$

$m^2 = \frac{9}{64}$

$m = \pm\frac{3}{8}$

Given that a = <2, -3>, b = <3, 4>, and c = <-1, 5>, find the number.

11. (a) $a \cdot (b + c)$ (b) $a \cdot b + a \cdot c$

(a) $= \langle 2,-3\rangle \cdot (\langle 3,4\rangle + \langle -1,5\rangle)$

$= \langle 2,-3\rangle \cdot \langle 2,9\rangle$

$= 4 - 27$

$= -23$

(b) $\langle 2,-3\rangle \cdot \langle 3,4\rangle + \langle 2,-3\rangle \cdot \langle -1,5\rangle$

$(6-12) + (-2-15)$

$= -23$

12. $(2a + b) \cdot (3c)$ (b) $(a - b) \cdot (b + c)$

$(\langle 2,-3\rangle - \langle 3,4\rangle) \cdot (\langle 3,4\rangle + \langle -1,5\rangle)$

$(\langle 4,-6\rangle + \langle 3,4\rangle) \cdot \langle -3,15\rangle$

$= \langle 7,-2\rangle \cdot \langle -3,15\rangle$

$= -21 - 30$

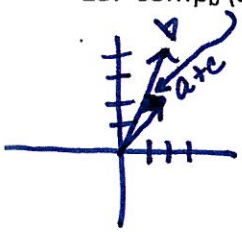
$= -51$

$= \langle -1,-7\rangle \cdot \langle 2,9\rangle$

$= -2 - 63$

$= -65$

13. $\text{comp}_b (a + c) = \frac{(a+c) \cdot b}{\|b\|}$

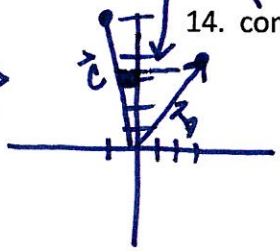


$\frac{(\langle 2,-3\rangle + \langle -1,5\rangle) \cdot \langle 3,4\rangle}{\sqrt{9+16}}$

$\frac{\langle 1,2\rangle \cdot \langle 3,4\rangle}{5}$

$= \frac{3+8}{5} = \frac{11}{5} = 2.2$

14. $\text{comp}_c b$



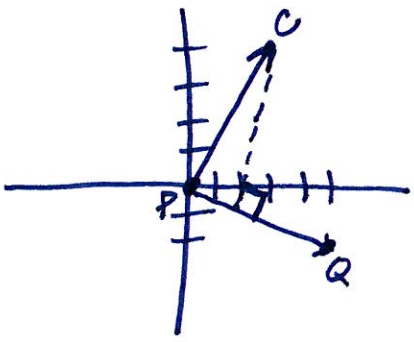
$= \frac{b \cdot c}{\|c\|}$

$= \frac{\langle 3,4\rangle \cdot \langle -1,5\rangle}{\sqrt{1+25}}$

$= \frac{-3+20}{\sqrt{26}} = \frac{17}{\sqrt{26}} \approx 3.33$

If c represents a constant force, find the work done if the point of application of c moves along the line segment from P to Q.

15. $c = 3i + 4j$; P(0,0), Q(5,-2)



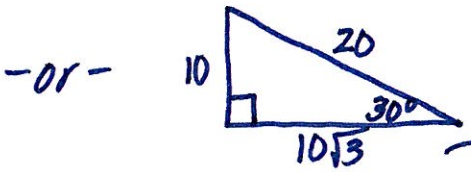
$W = \vec{c} \cdot \vec{PQ}$ = (Force)*(Distance)

$= \langle 3,4\rangle \cdot \langle 5,-2\rangle$

$= 15 + -8$

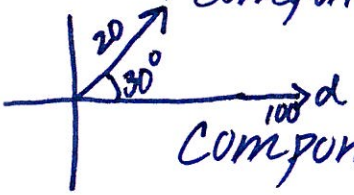
$= 7$

-or-



$10\sqrt{3} \cdot 100 = 1000\sqrt{3} = 1732$ ft.-lb.

16. A child pulls a wagon along level ground by exerting a force of 20 pounds on a handle that makes an angle of 30° with the horizontal. Find the work done in pulling the wagon 100 feet.



components of force vector $(20\cos 30°, 20\sin 30°)$

$= (10\sqrt{3}, 10)$

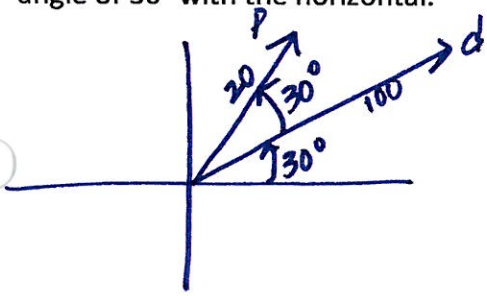
components of direction vector $(100, 0)$

(distance)

Work done $= F * D$

$= \langle 10\sqrt{3}, 10 \rangle \cdot \langle 100, 0 \rangle = 1000\sqrt{3} \approx 1732$ ft/lb

17. Find the work done if the wagon is pulled, with the same force, 100 feet up an incline that makes an angle of 30° with the horizontal.



force vector $(20\cos 60°, 20\sin 60°)$

$= (10, 10\sqrt{3})$

direction vector $= (100\cos 30, 100\sin 30)$

$(50\sqrt{3}, 50)$

Work done $= \langle 10, 10\sqrt{3} \rangle \cdot \langle 50\sqrt{3}, 50 \rangle$

$= 500\sqrt{3} + 500\sqrt{3} = 1000\sqrt{3} \approx 1732$ ft/lb

18. The sun has a radius of 432,000 miles, and its center is 93,000,000 miles from the center of Earth. Let v and w be the vectors illustrated in the figure.

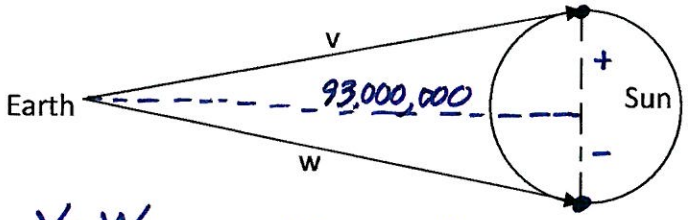
horizontal magnitude = 93,000,000

vertical magnitude = 432,000

(a) Express v and w in **i**, **j** form.

$V = 93{,}000{,}000i + 432{,}000j$

$W = 93{,}000{,}000i - 432{,}000j$



(b) Approximate the angle between v and w.

$$\cos\theta = \frac{V \cdot W}{\|V\| \cdot \|W\|} = \frac{93{,}000{,}000^2 - 432{,}000^2}{\left(\sqrt{93{,}000{,}000^2 + 432{,}000^2}\right)^2}$$

$\cos\theta \approx 0.99995845$

$\theta \approx 0.53°$