*AAT (IC/HW)-Days 1-2 Name Key

Chapter 8: 8-3 Vectors Date: ________ Period: ________

Find a + b, a - b, 4a + 5b, 4a - 5b, and ||a||.

1. a = <2, -3>, b = <1, 4>

$a+b=\langle 3,1\rangle$

$a-b=\langle 1,-7\rangle$

$4a+5b=\langle 8,-12\rangle+\langle 5,20\rangle$

$=\langle 13,8\rangle$

$4a-5b=\langle 3,-32\rangle$

$\|a\|=\sqrt{2^2+(-3)^2}=\sqrt{13}$

2. a = -<7, -2>, b = 4<-2, 1>

$a=\langle -7,2\rangle, b=\langle -8,4\rangle$

$a+b=\langle -15,6\rangle$

$a-b=\langle 1,-2\rangle$

$4a+5b=\langle -28,8\rangle+\langle -40,20\rangle$

$=\langle -68,28\rangle$

$4a-5b=\langle 12,-12\rangle$

$\|a\|=\sqrt{(-7)^2+(2)^2}$

$=\sqrt{53}$

3. a = i + 2j, b = 3i - 5j

$a+b=4i-3j$

$a-b=-2i+7j$

$4a+5b=4i+8j+15i-25j$

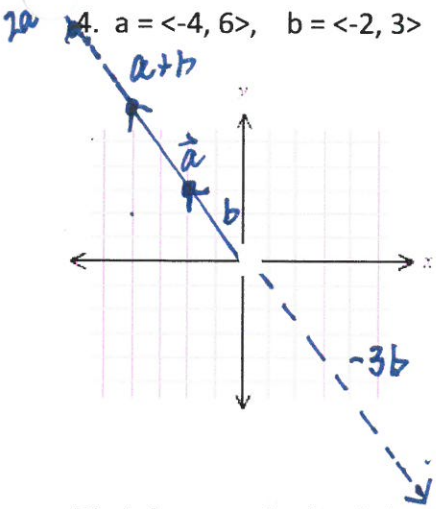
$=19i-17j$

$4a-5b=-11i+33j$

$\|a\|=\sqrt{1^2+2^2}=\sqrt{5}$

Sketch vectors corresponding to a, b, a+b, 2a, and -3b.

4. a = <-4, 6>, b = <-2, 3>

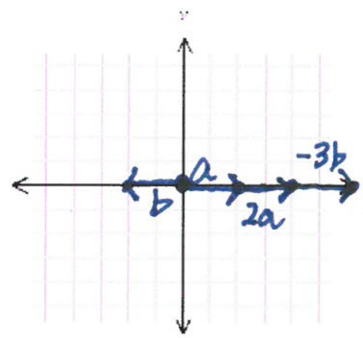


$a+b=\langle -6,9\rangle$

$2a=\langle -8,12\rangle$

$-3b=\langle 6,-9\rangle$

5. **a = <2, 0>, b = <-2, 0>**



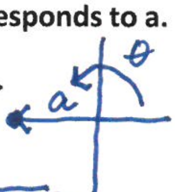
$a+b=\langle 0,0\rangle$

$2a=\langle 4,0\rangle$

$-3b=\langle 6,0\rangle$

Find the magnitude of the vector a and the smallest positive angle θ from the positive x-axis to the vector OP that corresponds to a.

6. **a** = <-5, 0>

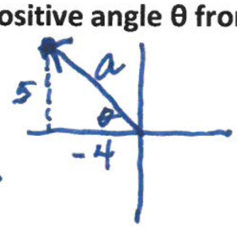


$\|a\|=\sqrt{(-5)^2+(0)^2}$

$=\sqrt{25}=5$

$\theta=\pi$ or $180°$

7. **a** = -4i + 5j



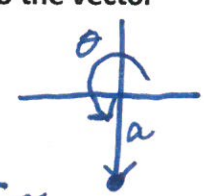
$\|a\|=\sqrt{16+25}$

$=\sqrt{41}$

$\tan^{-1}\left(-\frac{5}{4}\right)\approx -51.3°$

$\theta=180-51.3=128.7°$

8. **a** = -18j



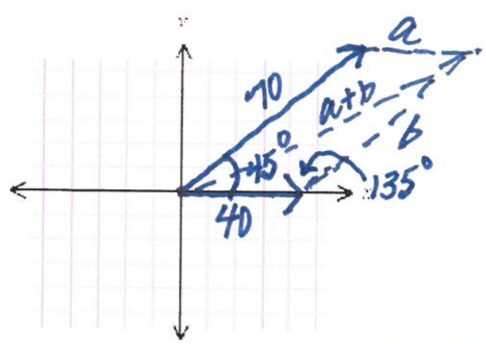
$\|a\|=\sqrt{0^2+(-18)^2}$

$=18$

$\theta=\frac{3\pi}{2}$ or $270°$

The vectors a and b represent two forces acting at the same point, and θ is the smallest positive angle between a and b. Approximate the magnitude of the resultant force.

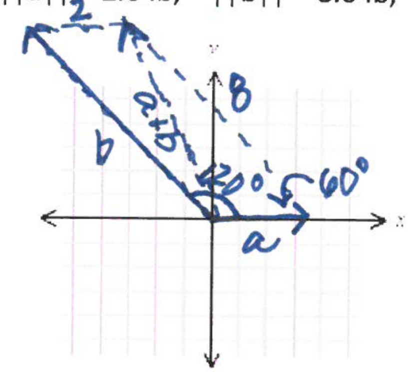
9. $||a|| = 40$ lb, $||b|| = 70$ lb, $\theta = 45°$



$\|a+b\| = \sqrt{40^2 + 70^2 - 2(40)(70)\cos 135°}$

≈ 102 lb

10. $||a|| = 2.0$ lb, $||b|| = 8.0$ lb, $\theta = 120°$

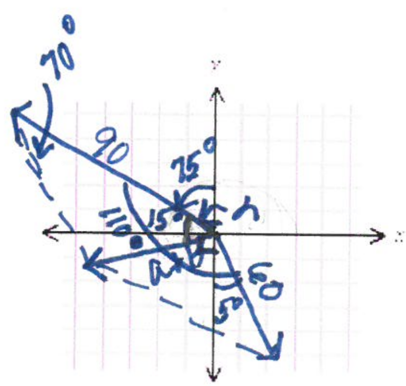


$\|a+b\| = \sqrt{2^2 + 8^2 - 2(2)(8)\cos 60°}$

≈ 7.2 lb.

The magnitudes and directions of two forces acting at a point P are given in (a) and (b). Approximate the magnitude and direction of the resultant vector.

11. (a) 90 lb, N75°W (b) 60 lb, S5°E



$\|a+b\| = \sqrt{90^2 + 60^2 - 2(90)(60)\cos 70°}$

≈ 89 lb

$\frac{\sin \alpha}{60} = \frac{\sin 70}{89}$

$\alpha \approx 39°$

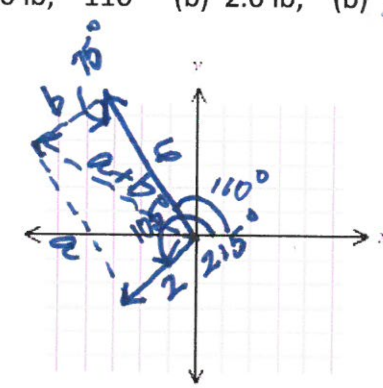
$\theta = 90° + 75° + 39° = 204°$

-or- S66°W

$39° - 15° = 24°$

$90° - 24° = 66°$

12. (a) 6.0 lb, 110° (b) 2.0 lb, (b) 215°



$\|a+b\| = \sqrt{6^2 + 2^2 - 2(6)(2)\cos 75°}$

≈ 5.8 lb.

$\frac{\sin \alpha}{2} = \frac{\sin 75}{5.8}$

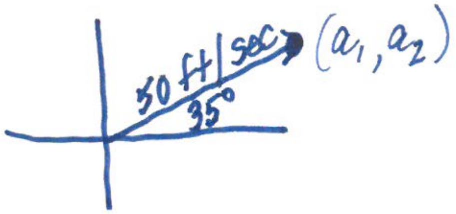
$\alpha \approx 19°$

$\theta = 110° + 19°$

$\approx 129°$

Approximate the horizontal and vertical components of the vector that is described.

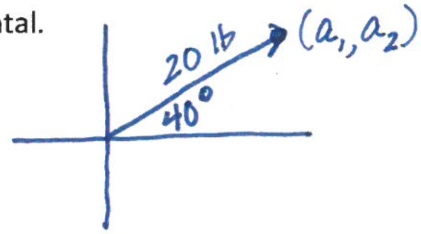
13. A quarterback releases a football with a speed of 50 ft/sec at an angle of 35° with the horizontal.



$a_1 = 50\cos 35°$ $= 40.96$

$a_2 = 50\sin 35$ $= 28.68$

14. A child pulls a sled through the snow by exerting a force of 20 pounds at an angle of 40° with the horizontal.



$a_1 = 20\cos 40°$ ≈ 15.32

$a_2 = 20\sin 40°$ ≈ 12.86

Find a unit vector that has (a) the same direction as the vector a and (b) the opposite direction of the vector a.

15. a = -8i + 15j

$\frac{1}{\|a\|}\cdot a$

$\|a\| = \sqrt{64+225}$
$= \sqrt{289} = 17$

$\frac{1}{17}(-8i + 15j)$

(a) $= -\frac{8}{17}i + \frac{15}{17}j$

(b) $= \frac{8}{17}i - \frac{15}{17}j$

16. a = <2, -5>

$\frac{1}{\|a\|}\cdot a$

$\|a\| = \sqrt{4+25} = \sqrt{29}$

$\frac{1}{\sqrt{29}}\cdot\langle 2,-5\rangle$

(a) $= \frac{2}{\sqrt{29}}, -\frac{5}{\sqrt{29}}$

(b) $= \frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}}$

17. Find a vector that has the same direction as <-6, 3> and

(a) twice the magnitude

$2(\langle -6,3\rangle) = \langle -12, 6\rangle$

(b) one-half the magnitude

$\frac{1}{2}(\langle -6,3\rangle) = \langle -3, 3/2\rangle$

Unit Vector $\frac{1}{\|a\|} \cdot a$

18. Find a vector of magnitude 6 that has the opposite direction of a = 4i - 7j.

$\|a\| = \sqrt{16+49} = \sqrt{65}$

Same direction: $\frac{4i}{\sqrt{65}} - \frac{7j}{\sqrt{65}}$

Opp. direction: $\frac{-4i}{\sqrt{65}} + \frac{7j}{\sqrt{65}} \xrightarrow{\times 6} \frac{-24i}{\sqrt{65}} + \frac{42j}{\sqrt{65}}$

19. If forces F_1, F_2, ...F_n act at a point P, the net (or resultant) force F is the sum $F_1 + F_2 + ... + F_n$. If F = 0, the forces are said are said to be in equilibrium. The given forces act at the origin O of an xy-plane.

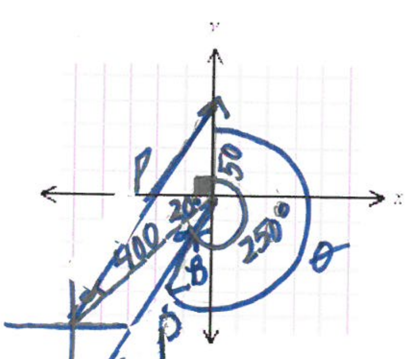
(a) Find the net force F. (b) Find an additional force G such that equilibrium occurs.

$F_1 = \langle -3, -1 \rangle$, $F_2 = \langle 0, -3 \rangle$, $F_3 = \langle 3, 4 \rangle$

(a) net force $= F_1 + F_2 + F_3 = \langle 0, 0 \rangle$

(b) No additional force is needed since system is in equilibrium.

20. An airplane pilot wishes to maintain a true course in the direction of 250° with a ground speed of 400 mi/hr when the wind is blowing directly north at 50 mi/hr. Approximate the required airspeed and compass heading.



$\|p\| = \sqrt{400^2 + 50^2 - 2(400)(50)\cos 110^\circ}$

≈ 420 mi/hr.

$\frac{\sin\alpha}{50} = \frac{\sin 110}{420}$

$\alpha = 6.42$; $\beta = 70 - 6.42$
$= 63.58^\circ$

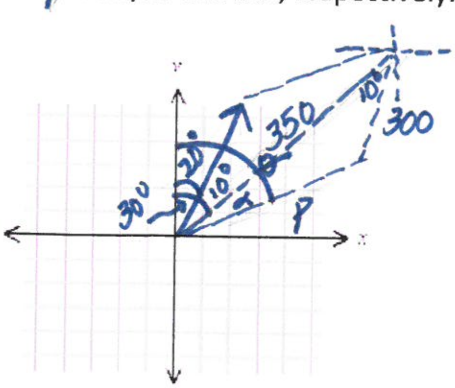
directional form

$\theta = 180^\circ + 63.58^\circ = 244^\circ$

or

$180^\circ + 20^\circ + 6.42^\circ$
$\approx 206^\circ$ from x-axis

21. An airplane is flying in the direction 20° with an airspeed of 300 mi/hr. Its ground speed and true course are 350 mi/hr and 30°, respectively. Approximate the direction and speed of the wind.



$\|p\| = \sqrt{300^2 + 350^2 - 2(300)(350)\cos 10}$

$P = 75$ mi/hr.

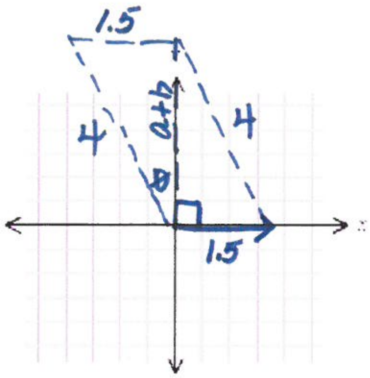
$\frac{\sin\alpha}{300} = \frac{\sin 10}{75}$

$\alpha \approx 44^\circ$

$\theta = 44^\circ + 30^\circ$

$\theta = 74^\circ$

22. The current in a river flows directly from the west at a rate of 1.5 ft/sec. A person who rows a boat at a rate of 4 ft/sec in still water wishes to row directly north across the river. Approximate, to the nearest degree, the direction in which the person should row.



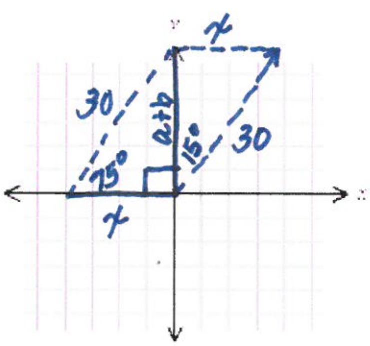
$$\|a+b\| = \sqrt{4^2 - 1.5^2}$$
$$\approx 3.7$$

$$\cos\theta = \frac{1.5^2 - 4^2 - 3.7^2}{(-2 \cdot 4 \cdot 3.7)}$$

$$\theta = 22^\circ$$

N22°W -or- $90^\circ + 22^\circ = 112^\circ$

23. For a motorboat moving at a speed of 30 mi/hr to travel directly north across a river, it must aim at a point that has the bearing N15°E. If the current is flowing directly west, approximate the rate at which it flows.



$$\sin 75 = \frac{\vec{a}+\vec{b}}{30}$$

$$\vec{a}+\vec{b} = 28.98$$

-or- $\sin 15 = \frac{x}{30}$

$x \approx 7.76$ or 8 mi/h

$$\frac{\sin 75}{28.98} = \frac{\sin 15}{x}$$

$x \approx 7.76$ or 8 mi/hr