

Chapter 7: 7-6 The Inverse Trigonometric Functions

Find the exact value of the expression whenever it is defined. *→ it is essential to choose the value y in the range*

1. (a) $\sin^{-1}(-\frac{\sqrt{2}}{2})$

$$-\frac{\pi}{4}$$

(b) $\cos^{-1}(-\frac{1}{2})$

$$\frac{2\pi}{3}$$

i.e.
 $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

Not $\frac{5\pi}{6}$ b/c
not in range
 $[-\frac{\pi}{2}, \frac{\pi}{2}]$

(c) $\tan^{-1}(-\sqrt{3})$

$$-\frac{\pi}{3}$$

2. (a) $\arcsin(\frac{\sqrt{3}}{2})$

$$\frac{\pi}{3}$$

(b) $\arccos\frac{\sqrt{2}}{2}$

$$\frac{\pi}{4}$$

(c) $\arctan\frac{1}{\sqrt{3}}$

$$\frac{\pi}{6}$$

 \cos^{-1}
(1) $\cos(\cos^{-1}x) = x$ if $-1 \leq x \leq 1$
(2) $\cos^{-1}(\cos y) = y$ if $0 \leq y \leq \pi$

3. (a) $\sin^{-1}\frac{\pi}{3}$

not defined

$$\frac{\pi}{3} > 1 \quad [-1 \leq x \leq 1]$$

(b) $\cos^{-1}\frac{\pi}{2}$

not defined

$$\frac{\pi}{2} > 1 \quad [-1 \leq x \leq 1]$$

(c) $\tan^{-1} 1$

$$\frac{\pi}{4}$$

4. (a) $\sin[\arcsin(-\frac{3}{10})]$

$$-\frac{3}{10}$$

$$-1 \leq -\frac{3}{10} \leq 1$$

(b) $\cos(\arccos\frac{1}{2})$

$$\frac{1}{2}$$

$$-1 \leq \frac{1}{2} \leq 1$$

(c) $\tan(\arctan 14)$

$$14$$

 $\tan(\arctan x) = x$ for every x .

5. (a) $\sin^{-1}(\sin \frac{\pi}{3})$

$$\frac{\pi}{3}$$

$$-\frac{\pi}{2} \leq \frac{\pi}{3} \leq \frac{\pi}{2}$$

(b) $\cos^{-1}[\cos(\frac{5\pi}{6})]$

$$\frac{5\pi}{6}$$

$$0 \leq \frac{5\pi}{6} \leq \pi$$

(c) $\tan^{-1}[\tan(-\frac{\pi}{6})]$

$$-\frac{\pi}{6}$$

$$-\frac{\pi}{2} < -\frac{\pi}{6} < \frac{\pi}{2}$$

6. (a) $\arcsin(\sin \frac{5\pi}{4})$

$$\arcsin(-\frac{\sqrt{2}}{2})$$

$$= -\frac{\pi}{4}$$

(b) $\arccos(\cos \frac{5\pi}{4})$

$$\arccos(-\frac{\sqrt{2}}{2})$$

$$=\frac{3\pi}{4}$$

(c) $\arctan(\tan \frac{7\pi}{4})$

$$\arctan(-1)$$

$$= -\frac{\pi}{4}$$

not in range
 $(-\frac{\pi}{2}, \frac{\pi}{2})$
 \tan^{-1}
(1) $\tan(\tan^{-1}x) = x$ for every x
(2) $\tan^{-1}(\tan y) = y$ if $-\frac{\pi}{2} < y < \frac{\pi}{2}$

$$7. (a) \sin [\cos^{-1}(-\frac{1}{2})]$$

$$\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$$

$$(b) \cos (\tan^{-1} 1)$$

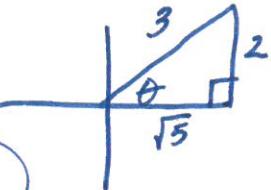
$$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$(c) \tan [\sin^{-1} (-1)]$$

$$\tan(-\frac{\pi}{2}) \text{ undefined}$$

$$8. (a) \cot(\sin^{-1} \frac{2}{3})$$

$$\cot \theta = \frac{15}{2}$$



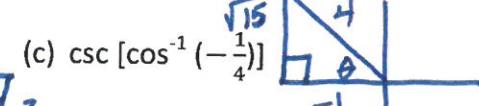
$$(b) \sec[\tan^{-1}(-\frac{3}{5})]$$

$$\sec \theta = \frac{\sqrt{34}}{5}$$

$$5$$

$$-3$$

$$\sqrt{34}$$



$$\csc \theta = \frac{4}{\sqrt{15}} = \frac{4\sqrt{15}}{15}$$

$$9. (a) \sin(\arcsin \frac{1}{2} + \arccos 0)$$

$$\sin(\frac{\pi}{6} + \frac{\pi}{2})$$

$$= \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} & \text{or } \sin(\alpha + \beta) \\ & = \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ & = \sin \frac{\pi}{6} \cos \frac{\pi}{2} + \cos \frac{\pi}{6} \sin \frac{\pi}{2} \\ & = 0 + \frac{\sqrt{3}}{2} \\ & = \frac{\sqrt{3}}{2} \end{aligned}$$

$$(b) \cos(\arctan(-\frac{3}{4}) - \arcsin \frac{4}{5})$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\frac{4}{5} \cdot \frac{3}{5}$$

$$+ (-\frac{3}{5})(\frac{4}{5})$$

$$= 0$$

$$(c) \tan(\arctan \frac{4}{3} + \arccos \frac{8}{17})$$

$$\tan(\alpha + \beta) =$$

$$\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

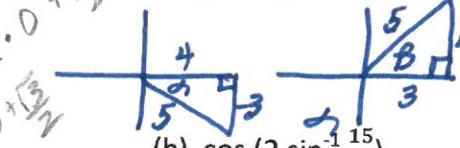
$$10. (a) \sin[2 \arccos(-\frac{3}{5})]$$

$$\sin 2\alpha =$$

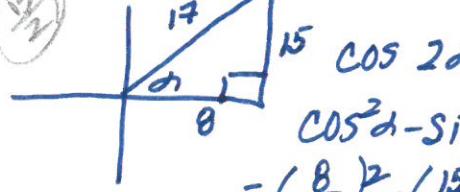
$$= 2 \sin \alpha \cos \alpha$$

$$= 2(\frac{4}{5})(-\frac{3}{5})$$

$$= -\frac{24}{25}$$



$$(b) \cos(2 \sin^{-1} \frac{15}{17})$$

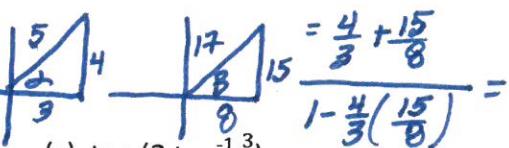


$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$= (\frac{8}{17})^2 - (\frac{15}{17})^2$$

$$= -\frac{161}{289}$$

$$= -\frac{161}{289}$$



$$(c) \tan(2 \tan^{-1} \frac{3}{4})$$

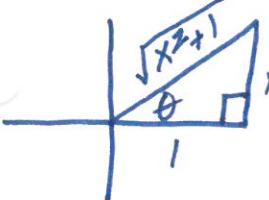
$$= \frac{77}{24}$$

$$- \frac{3}{2}$$

$$= \frac{-77}{36}$$

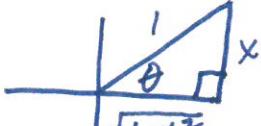
Write the expression as an algebraic expression in x for x > 0.

$$11. \sin(\tan^{-1} x)$$



$$\sin \theta = \frac{x}{\sqrt{x^2 + 1}}$$

$$12. \sin(2 \sin^{-1} x)$$

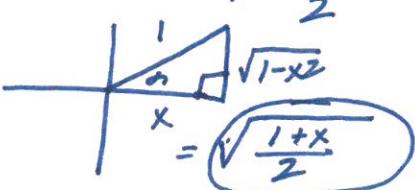


$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2x \sqrt{1-x^2}$$

$$13. \cos(\frac{1}{2} \arccos x)$$

$$\cos \frac{1}{2} \alpha = \sqrt{\frac{1+\cos \alpha}{2}}$$



$$= \sqrt{\frac{1+x}{2}}$$