***AAT (IC/HW)-Day 1** Name Key

Chapter 7: 7-5 Product-to-Sum and Sum-to-Product Formulas Date: _____ Period: _____

Express as a sum or difference.

1. sin 7t sin 3t

$= \frac{1}{2}(\cos(7t-3t)-\cos(7t+3t))$

$= \frac{1}{2}\cos 4t - \frac{1}{2}\cos 10t$

2. cos 6u cos (-4u)

$\frac{1}{2}(\cos(6u+(-4u))+\cos(6u-(-4u)))$

$\frac{1}{2}\cos 2u + \frac{1}{2}\cos 10u$

3. 2 sin 9θ cos 3θ

$\sin(9\theta+3\theta)+\sin(9\theta-3\theta)$

$\sin 12\theta + \sin 6\theta$

4. 3 cos x sin 2x

$\frac{3}{2}(\sin(x+2x)-\sin(x-2x))$

$\frac{3}{2}\sin 3x - \frac{3}{2}\sin(-x)$

$\frac{3}{2}\sin 3x + \frac{3}{2}\sin x$

Express as a product.

5. sin 6θ + sin 2θ

$2\sin\frac{6\theta+2\theta}{2}\cos\frac{6\theta-2\theta}{2}$

$2\sin 4\theta\cos 2\theta$

6. cos 5x - cos 3x

$-2\sin\frac{5x+3x}{2}\sin\frac{5x-3x}{2}$

$= -2\sin 4x \sin x$

7. sin 3t - sin 7t

$2\cos\frac{3t+7t}{2}\sin\frac{3t-7t}{2}$

$2\cos 5t \sin(-2t)$

$-2\cos 5t \sin 2t$

8. cos x + cos 2x

$2\cos\frac{x+2x}{2}\cos\frac{x-2x}{2}$

$2\cos\frac{3}{2}x\cos\left(-\frac{1}{2}x\right)$

$2\cos\frac{3}{2}x\cos\frac{1}{2}x$

Verify the identity.

9. $\frac{\sin 4t+\sin 6t}{\cos 4t-\cos 6t} = \cot t$

$\frac{2\sin 5t\cos(-t)}{2\sin 5t\sin(-t)}$

$\frac{\cos t}{\sin t}$

$\cot t$

10. $\frac{\sin\theta+\sin 3\theta}{\cos\theta+\cos 3\theta} = \tan 2\theta$

$\frac{2\sin 2\theta\cos(-\theta)}{2\cos 2\theta\cos(-\theta)}$

$= \frac{\sin 2\theta}{\cos 2\theta}$

$= \tan 2\theta$

11. $\frac{\sin u+\sin v}{\cos u+\cos v} = \tan\frac{1}{2}(u+v)$

$\frac{2\sin\frac{1}{2}(u+v)\cos\frac{1}{2}(u-v)}{2\cos\frac{1}{2}(u+v)\cos\frac{1}{2}(u-v)}$

$= \frac{\sin\frac{1}{2}(u+v)}{\cos\frac{1}{2}(u+v)}$

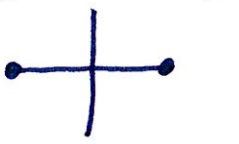
$= \tan\frac{1}{2}(u+v)$

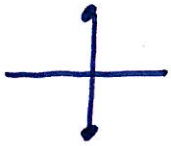
Use sum-to-product formulas to find the solutions of the equation.

12. $\sin 5t + \sin 3t = 0$

$\frac{2}{2}\sin 4t \cos t = \frac{0}{2}$

$(\sin 4t = 0)$ $(\cos t = 0)$





$4t = \pi n$ $t = \frac{\pi}{2} + \pi n$

$t = \frac{\pi}{4} n$ ← included

13. $\cos x = \cos 3x$

$\cos x - \cos 3x = 0$

$-2\sin 2x \sin(-x) = 0$

$\frac{2}{2}\sin 2x \sin x = \frac{0}{2}$

$(\sin 2x = 0)$ $(\sin x = 0)$

$\frac{2x}{2} = \frac{\pi n}{2}$ $x = \pi n$

$x = \frac{\pi}{2} n$ ← included

14. $\cos 3x + \cos 5x = \cos x$

$(\cos 3x + \cos 5x) - \cos x = 0$

$2\cos 4x \cos x - \cos x = 0$

$\cos x (2\cos 4x - 1) = 0$

$\cos x = 0$; $\cos 4x = \frac{1}{2}$

$x = \frac{\pi}{2} + \pi n$

$\frac{4x}{4} = \frac{\frac{\pi}{3} + 2\pi n}{4}$

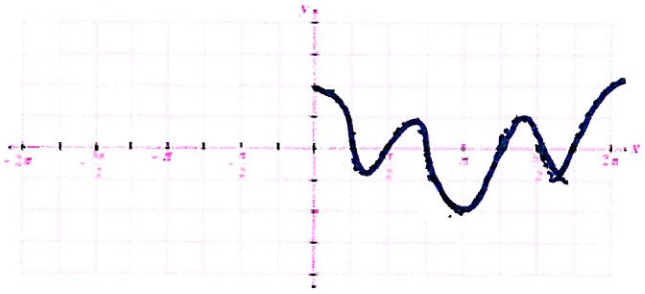
$\frac{4x}{4} = \frac{\frac{5\pi}{3} + 2\pi n}{4}$

$x = \frac{\pi}{12} + \frac{\pi}{2} n$

$x = \frac{5\pi}{12} + \frac{\pi}{2} n$

Graph the function f for $0 \le x \le 2\pi$. Use a sum-to-product formula to help find the x-intercepts.

15. $f(x) = \cos x + \cos 3x$



$= 2\cos 2x \cos(-x) = 0$

$= \frac{2}{2}\cos 2x \cos x = \frac{0}{2}$

$(\cos 2x = 0)$ $(\cos x = 0)$

$\frac{2x}{2} = \frac{\frac{\pi}{2} + \pi n}{2}$ $x = \frac{\pi}{2} + \pi n$

$x = \frac{\pi}{4} + \frac{\pi}{2} n$

$n =$	0	1	2	3	0	1
$x =$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{7\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{2}$