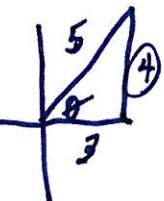


Find the exact values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$ for the given values of θ .

1. $\cos \theta = \frac{3}{5}$; $0^\circ < \theta < 90^\circ$

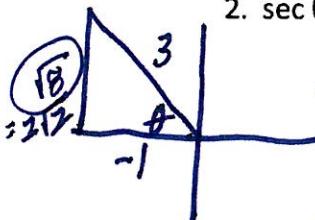


$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ &= 2 \left(\frac{4}{5}\right) \left(\frac{3}{5}\right) \\ &= \frac{24}{25}\end{aligned}$$

$$\begin{aligned}\cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= \left(\frac{9}{25}\right) - \left(\frac{16}{25}\right) \\ &= -\frac{7}{25}\end{aligned}$$

$$\begin{aligned}\tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta} \\ &= \frac{2 \left(\frac{4}{3}\right)}{1 - \left(\frac{16}{9}\right)} = \frac{\frac{8}{3}}{-\frac{7}{9}} = -\frac{24}{7}\end{aligned}$$

2. $\sec \theta = -3$; $90^\circ < \theta < 180^\circ$



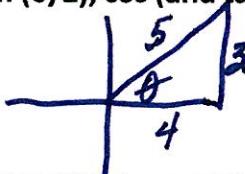
$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ &= 2 \left(\frac{2\sqrt{2}}{3}\right) \left(-\frac{1}{3}\right) = -\frac{4\sqrt{2}}{9}\end{aligned}$$

$$\begin{aligned}\cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= \left(-\frac{1}{3}\right)^2 - \left(\frac{2\sqrt{2}}{3}\right)^2 \\ &= \frac{1}{9} - \frac{8}{9} = -\frac{7}{9} \\ \tan 2\theta &= \frac{\sin 2\theta}{\cos 2\theta} = \frac{-\frac{4\sqrt{2}}{9}}{-\frac{7}{9}} = \frac{4\sqrt{2}}{7}\end{aligned}$$

Find the exact values of $\sin(\theta/2)$, \cos (and \tan) $(\theta/2)$ for $\theta/2$, the given conditions.

3. $\sec \theta = \frac{5}{4}$; $0^\circ < \theta < 90^\circ$

$$\cos \theta = \frac{4}{5}$$



$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{1 - \frac{4}{5}}{2}} = \sqrt{\frac{\frac{1}{5}}{2}} = \sqrt{\frac{1}{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} = \sqrt{\frac{1 + \frac{4}{5}}{2}} = \sqrt{\frac{9/5}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\tan \frac{\theta}{2} = \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} = \frac{\frac{\sqrt{10}}{10}}{\frac{3\sqrt{10}}{10}} = \frac{1}{3}$$

4. $\tan \theta = 1$; $-180^\circ < \theta < -90^\circ$



QIV: $-90^\circ < \frac{\theta}{2} < -45^\circ$

$$\sin \frac{\theta}{2} = -\sqrt{\frac{1 - \cos \theta}{2}} = -\sqrt{\frac{1 + \frac{1}{2}}{2}} = -\frac{\sqrt{3}}{2}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} = \sqrt{\frac{1 + \frac{1}{2}}{2}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta} = \frac{1 + \frac{1}{2}}{\frac{\sqrt{2 + \sqrt{2}}}{2}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

Use half-angle formulas to find the exact values.

5. (a) $\cos 67^\circ 30'$

$$\sqrt{\frac{1 + \cos 135^\circ}{2}} = \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}}$$

$$= \sqrt{\frac{2 - \sqrt{2}}{4}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$\sqrt{\frac{1 - \cos 30^\circ}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{2 - \sqrt{3}}{4}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$$

(b) $\sin 15^\circ$

$$\frac{1 - \cos \frac{3\pi}{4}}{\sin \frac{3\pi}{4}} = \frac{1 + \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

$$= \frac{2 + \sqrt{2}}{\sqrt{2}} = \sqrt{2 + \sqrt{2}}$$

(c) $\tan \frac{3\pi}{8}$

$$-\frac{\sqrt{2}}{2} = -\sqrt{2 - 1}$$

* rationalize

Verify the identity.

6. $\sin 10\theta = 2 \sin 5\theta \cos 5\theta$

$$\sin 10\theta =$$

$$\sin 2(5\theta) = 2 \sin 5\theta \cos 5\theta$$

$$2 \sin 5\theta \cos 5\theta = 2 \sin 5\theta \cos 5\theta$$

7. $4 \sin \frac{x}{2} \cos \frac{x}{2} = 2 \sin x$

$$2(2 \sin \frac{x}{2} \cos \frac{x}{2}) =$$

$$2 \sin \left(2 \left(\frac{x}{2} \right) \right)$$

$$2 \sin x = 2 \sin x$$

8. $(\sin t + \cos t)^2 = 1 + \sin 2t$

$$\underline{\sin^2 t + 2 \sin t \cos t + \cos^2 t} =$$

$$\underline{1 + \sin 2t} = 1 + \sin 2t$$

Find the solutions of the equation that are in the interval $[0, 2\pi]$.

9. $\underline{\sin 2t + \sin t = 0}$

$$\underline{2 \sin t \cos t + \sin t = 0}$$

$$\sin t(2 \cos t + 1) = 0$$

$$\sin t = 0 \quad \cos t = -\frac{1}{2}$$

$$t = 0, \pi; \frac{2\pi}{3}, \frac{4\pi}{3}$$

10. $\underline{\cos u + \cos 2u = 0}$

$$\underline{\cos u + 2 \cos^2 u - 1 = 0}$$

$$(2 \cos u - 1)(\cos u + 1) = 0$$

$$\cos u = \frac{1}{2}; \cos u = -1$$

$$u = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$$

11. $\tan 2x = \tan x$

$$\frac{\sin 2x}{\cos 2x} = \frac{\sin x}{\cos x}$$

$$\sin 2x \cos x = \sin x \cos 2x$$

$$\sin 2x \cos x - \sin x \cos 2x = 0$$

$$\sin(2x - x) = 0$$

$$\sin x = 0$$

$$x = 0, \pi$$

Use the graph of f to find the simplest expression $g(x)$ such that the equation $f(x) = g(x)$ is an identity.

Verify the identity.

12. $f(x) = \frac{\sin x (1 + \cos 2x)}{\sin 2x} = \frac{\sin x (1 + (2 \cos^2 x - 1))}{2 \sin x \cos x} = \frac{2 \cos^2 x}{2 \cos x} = \underline{\cos x}$

