

Express as a cofunction of a complementary angle.

1. (a) $\sin 46^\circ 37'$

$$\cos(90^\circ - 46^\circ 37')$$

$$= \cos 43^\circ 23'$$

(b) $\cos 73^\circ 12'$

$$\sin(90^\circ - 73^\circ 12')$$

$$= \sin 16^\circ 48'$$

(c) $\tan \frac{\pi}{6}$

$$\cot\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$$

$$= \cot \frac{2\pi}{6} = \cot \frac{\pi}{3}$$

(d) $\sec 17.28^\circ$

$$\csc(90^\circ - 17.28^\circ)$$

$$= \csc 72.72^\circ$$

2. (a) $\cos \frac{7\pi}{20}$

$$\sin\left(\frac{\pi}{2} - \frac{7\pi}{20}\right)$$

$$= \sin \frac{3\pi}{20}$$

(b) $\sin \frac{1}{4}$

$$\cos\left(\frac{\pi}{2} - \frac{1}{4}\right)$$

$$= \cos\left(\frac{2\pi - 1}{4}\right)$$

(c) $\tan 1$

$$\cot\left(\frac{\pi}{2} - 1\right)$$

$$= \cot\left(\frac{\pi - 2}{2}\right)$$

(d) $\csc 0.53$

$$\sec\left(\frac{\pi}{2} - .53\right)$$

Find the exact values.

3. (a) $\cos \frac{\pi}{4} + \cos \frac{\pi}{6}$

$$\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{2} + \sqrt{3}}{2}$$

(b) $\cos \frac{5\pi}{12}$ (use $\frac{5\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}$)

$$\cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

$$= \cos \frac{\pi}{4} \cos \frac{\pi}{6} - \sin \frac{\pi}{4} \sin \frac{\pi}{6}$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

4. (a) $\tan 60^\circ + \tan 225^\circ$

$$\sqrt{3} + 1$$

(b) $\tan 285^\circ$ (use $285^\circ = 60^\circ + 225^\circ$)

$$\frac{\tan 60 + \tan 225}{1 - \tan 60 \tan 225} = \frac{(\sqrt{3} + 1) \cdot (1 + \sqrt{3})}{(1 - \sqrt{3})(1 + \sqrt{3})}$$

$$= \frac{\sqrt{3} + 3 + 1 + \sqrt{3}}{1 - 3}$$

$$= \frac{2\sqrt{3} + 4}{-2} = -\sqrt{3} - 2$$

5. (a) $\sin \frac{3\pi}{4} - \sin \frac{\pi}{6}$

$$\frac{\sqrt{2}}{2} - \frac{1}{2}$$

(b) $\sin \frac{7\pi}{12}$ (use $\frac{7\pi}{12} = \frac{3\pi}{4} - \frac{\pi}{6}$)

$$\sin \frac{3\pi}{4} \cos \frac{\pi}{6} - \cos \frac{3\pi}{4} \sin \frac{\pi}{6}$$

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - -\frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

Express as a trigonometric function of one angle.

6. $\cos 48^\circ \cos 23^\circ + \sin 48^\circ \sin 23^\circ$

$$\begin{aligned} & \cos(48-23) \\ &= \cos 25^\circ \end{aligned}$$

7. $\cos 10^\circ \sin 5^\circ - \sin 10^\circ \cos 5^\circ$

$$\begin{aligned} & \sin(5-10) \\ &= \sin(-5^\circ) \end{aligned}$$

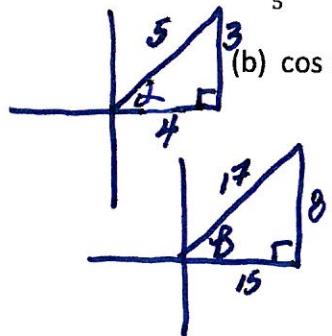
8. $\cos 3 \sin(-2) - \cos 2 \sin 3$

$$\begin{aligned} & \sin(-2-3) \\ &= \sin(-5) \end{aligned}$$

9. If α and β are acute angles such that $\cos \alpha = \frac{4}{5}$ and $\tan \beta = \frac{8}{15}$, find

(a) $\sin(\alpha + \beta)$

$$\begin{aligned} & \sin^2 \cos B + \cos \alpha \sin B \\ & : \frac{3}{5} \left(\frac{15}{17} \right) + \frac{4}{5} \left(\frac{8}{17} \right) \\ &= \frac{45}{85} + \frac{32}{85} = \frac{77}{85} \end{aligned}$$



(b) $\cos(\alpha + \beta)$

$$\begin{aligned} & \cos^2 \cos B - \sin \alpha \sin B \\ & : \frac{4}{5} \cdot \frac{15}{17} - \frac{3}{5} \cdot \frac{8}{17} \\ &= \frac{36}{85} \end{aligned}$$

(c) the quadrant containing $(\alpha + \beta)$

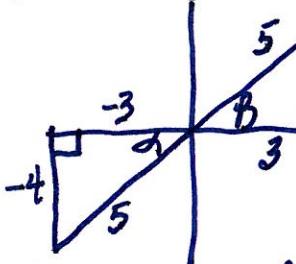
$\cos(+)$
 $\sin(+)$

I Quad

10. If $\sin \alpha = -\frac{4}{5}$ and $\sec \beta = \frac{5}{3}$ for a third-quadrant angle α and a first-quadrant angle β , find

(a) $\sin(\alpha + \beta)$

$$\begin{aligned} & \sin \alpha \cos B + \cos \alpha \sin B \\ & -\frac{4}{5} \left(\frac{3}{5} \right) + \left(-\frac{3}{5} \right) \left(\frac{4}{5} \right) \\ & -\frac{12}{25} + -\frac{12}{25} = -\frac{24}{25} \end{aligned}$$



(b) $\tan(\alpha + \beta)$

$$\begin{aligned} & \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \\ & \frac{-\frac{4}{3} + \frac{4}{3}}{1 - \left(\frac{4}{3} \cdot \frac{4}{3} \right)} = \frac{0}{-\frac{7}{9}} \\ & = 0 \end{aligned}$$

(c) the quadrant containing $(\alpha + \beta)$

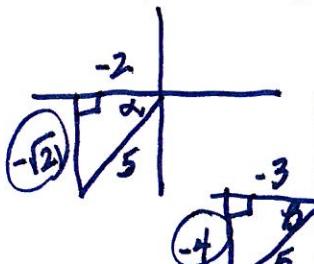
$\sin(-)$

$\tan(-)$

IV Quad

11. If α and β are third-quadrant angles such that $\cos \alpha = -\frac{2}{5}$ and $\cos \beta = -\frac{3}{5}$, find

(a) $\sin(\alpha - \beta)$



(b) $\cos(\alpha - \beta)$

$$\begin{aligned} & \sin \alpha \cos B - \cos \alpha \sin B \\ & \left(-\frac{\sqrt{21}}{5} \right) \left(-\frac{3}{5} \right) - \left(-\frac{2}{5} \right) \left(-\frac{4}{5} \right) \\ & \frac{3\sqrt{21}}{25} - \frac{8}{25} \approx 0.23 \end{aligned}$$

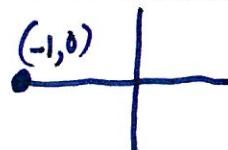
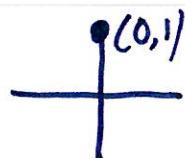
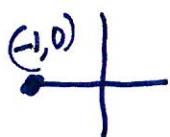
(c) the quadrant containing $(\alpha - \beta)$

$\sin(+)$

$\cos(+)$

Q1

Verify the reduction formula.



$$12. \sin(\theta + \pi) = -\sin \theta$$

$$\begin{aligned} \sin \theta \cos \pi + \cos \theta \sin \pi &= \\ \sin \theta \cdot (-1) + \cos \theta \cdot (0) &= \\ -\sin \theta &= -\sin \theta \end{aligned}$$

$$13. \sin(x - \frac{5\pi}{2}) = -\cos x$$

$$\begin{aligned} \sin x \cos \frac{5\pi}{2} - \cos x \sin \frac{5\pi}{2} &= \\ \sin x(0) - \cos x(1) &= \\ -\cos x &= -\cos x \end{aligned}$$

$$14. \cos(\theta - \pi) = -\cos \theta$$

$$\begin{aligned} \cos \theta \cos \pi + \sin \theta \sin \pi &= \\ \cos \theta \cdot (-1) + \sin \theta \cdot (0) &= \\ -\cos \theta &= -\cos \theta \end{aligned}$$

Verify the identity.

$$15. \sin(\theta + \frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\sin \theta + \cos \theta)$$

$$\begin{aligned} \sin \theta \cdot \cos \frac{\pi}{4} + \cos \theta \sin \frac{\pi}{4} &= \\ \sin \theta \cdot \frac{\sqrt{2}}{2} + \cos \theta \cdot \frac{\sqrt{2}}{2} &= \\ \frac{\sqrt{2}}{2}(\sin \theta + \cos \theta) &= \frac{\sqrt{2}}{2}(\sin \theta + \cos \theta) \end{aligned}$$

$$16. \tan(u + \frac{\pi}{4}) = \frac{1 + \tan u}{1 - \tan u}$$

$$\begin{aligned} \frac{\tan u + \tan \frac{\pi}{4}}{1 - \tan u \cdot \tan \frac{\pi}{4}} &= \\ \frac{\tan u + 1}{1 - \tan u} &= \frac{1 + \tan u}{1 - \tan u} = \frac{1 + \tan u}{1 - \tan u} \end{aligned}$$

Use the addition or subtraction formula to find the solutions of the equation that are in the interval $[0, \pi]$.

$$17. \sin 4t \cos t = \sin t \cos 4t$$

$$\sin 4t \cos t - \sin t \cos 4t = 0$$

$$\sin(4t - t) = 0$$

$$\sin(3t) = 0$$

$$\frac{3t}{3} = \frac{\pi n}{3}$$

$$t = \frac{\pi n}{3}; n=0, 1, 2$$

$$t = 0, \frac{\pi}{3}, \frac{2\pi}{3}$$

$$18. \cos 5t \cos 2t = -\sin 5t \sin 2t$$

$$\cos 5t \cos 2t + \sin 5t \sin 2t = 0$$

$$\cos(5t - 2t) = 0$$

$$\cos 3t = 0$$

$$3t = \frac{\pi}{2} + \pi n$$

$$t = \frac{\pi}{6} + \frac{\pi}{3}n; n=0, 1, 2$$

$$t = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$$