

*AAT (IC/HW)-Days 1 & 2

Chapter 7: 7-1 Verifying Trigonometric Identities

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Date: _____ Period: _____

Verify the identity.

1. $\csc \theta - \sin \theta = \cot \theta \cos \theta$

$$\frac{1}{\sin \theta} - \sin \theta =$$

$$\frac{1 - \sin^2 \theta}{\sin \theta} =$$

$$\frac{\cos^2 \theta}{\sin \theta} =$$

$$\frac{\cos \theta}{\sin \theta} \cdot \cos \theta = \cot \theta \cdot \cos \theta$$

3. $\frac{\csc^2 \theta}{1 + \tan^2 \theta} = \cot^2 \theta$

$$\frac{\csc^2 \theta}{\sec^2 \theta} =$$

$$\frac{1}{\sin^2 \theta} =$$

$$\frac{1}{\cos^2 \theta} = \cot^2 \theta$$

5. $\frac{1 + \cos 3t}{\sin 3t} + \frac{\sin 3t}{1 + \cos 3t} = 2 \csc 3t$

$$= \frac{(1 + \cos 3t)^2 + \sin^2 3t}{\sin 3t (1 + \cos 3t)} =$$

$$\frac{1 + 2 \cos 3t + \cos^2 3t + \sin^2 3t}{\sin 3t (1 + \cos 3t)} =$$

$$\frac{1 + 2 \cos 3t}{\sin 3t (1 + \cos 3t)} = \frac{2(1 + \cos 3t)}{\sin 3t (1 + \cos 3t)} = 2 \csc 3t$$

7. $(\sec u - \tan u)(\csc u + 1) = \cot u$

$$\left(\frac{1}{\cos u} - \frac{\sin u}{\cos u}\right)\left(\frac{1}{\sin u} + 1\right) =$$

$$\left(\frac{1 - \sin u}{\cos u}\right)\left(\frac{1 + \sin u}{\sin u}\right) =$$

$$\frac{1 - \sin^2 u}{\cos u \cdot \sin u} = \frac{\cos^2 u}{\cos u \cdot \sin u} = \frac{\cos u}{\sin u} =$$

$$\cot u$$

2. $\frac{\sec^2 2u - 1}{\sec^2 2u} = \sin^2 2u$

$$1 - \frac{1}{\sec^2 2u} =$$

$$1 - \cos^2 2u =$$

$$\sin^2 2u$$

4. $\tan t + 2 \cos t \csc t = \sec t \csc t + \cot t$

$$\frac{\sin t}{\cos t} + \frac{2 \cos t}{\sin t} =$$

$$\frac{\sin^2 t + 2 \cos^2 t}{\cos t \sin t} =$$

$$\frac{1 - \cos^2 t + 2 \cos^2 t}{\cos t \sin t} =$$

$$\frac{1 + \cos^2 t}{\cos t \sin t} = \frac{1}{\cos t \sin t} + \frac{\cos^2 t}{\sin t}$$

$$6. \frac{1}{1 - \cos y} + \frac{1}{1 + \cos y} = 2 \csc^2 y$$

$$\frac{1 + \cos y + 1 - \cos y}{1 - \cos^2 y} =$$

$$\frac{2}{\sin^2 y} =$$

$$2 \csc^2 y$$

8. $\csc^4 t - \cot^4 t = \csc^2 t + \cot^2 t$

$$(\csc^2 t - \cot^2 t)(\csc^2 t + \cot^2 t) =$$

$$(1)(\csc^2 t + \cot^2 t) =$$

$$\csc^2 t + \cot^2 t$$

$$9. \frac{\cos \beta}{1-\sin \beta} = \sec \beta + \tan \beta$$

$$\frac{\cos B}{1-\sin B} \cdot \frac{1+\sin B}{1+\sin B} =$$

$$\frac{\cos B(1+\sin B)}{1-\sin^2 B} =$$

$$\frac{\cos B(1+\sin B)}{\cos^2 B} = \frac{1+\sin B}{\cos B} = \frac{1}{\cos B} + \frac{\sin B}{\cos B} = \sec B + \tan B$$

$$11. \frac{\cot 4u-1}{\cot 4u+1} = \frac{1-\tan 4u}{1+\tan 4u}$$

$$\frac{\frac{1}{\tan 4u}-1}{\frac{1}{\tan 4u}+1} =$$

$$\frac{\frac{1-\tan 4u}{\tan 4u}}{\frac{1+\tan 4u}{\tan 4u}} = \frac{1-\tan 4u}{1+\tan 4u}$$

$$13. \tan^4 k - \sec^4 k = 1 - 2 \sec^2 k$$

$$(\tan^2 k - \sec^2 k)(\tan^2 k + \sec^2 k) =$$

$$(-1)(\sec^2 k - 1 + \sec^2 k) =$$

$$(-1)(2 \sec^2 k - 1)$$

$$1 - 2 \sec^2 k$$

$$15. (\sin^2 \theta + \cos^2 \theta)^3 = 1$$

$$(1)^3 =$$

$$(1)$$

$$10. \frac{\tan^2 x}{\sec x + 1} = \frac{1-\cos x}{\cos x}$$

$$\frac{\sec^2 x - 1}{\sec x + 1} =$$

$$\frac{(\sec x - 1)(\sec x + 1)}{(\sec x + 1)} =$$

$$\sec x - 1 =$$

$$\frac{1}{\cos x} - 1 = \frac{1 - \cos x}{\cos x}$$

$$12. \sin^4 r - \cos^4 r = \sin^2 r - \cos^2 r$$

$$(\sin^2 r - \cos^2 r)(\sin^2 r + \cos^2 r) =$$

$$(\sin^2 r - \cos^2 r)(1) =$$

$$\sin^2 r - \cos^2 r$$

$$14. (\sec t + \tan t)^2 = \frac{1+\sin t}{1-\sin t}$$

$$\left(\frac{1}{\cos t} + \frac{\sin t}{\cos t}\right)^2 =$$

$$\left(\frac{1+\sin t}{\cos t}\right)^2 =$$

$$\frac{(1+\sin t)^2}{(\cos^2 t)} = \frac{(1+\sin t)^2}{(1-\sin^2 t)} = \frac{(1+\sin t)^2}{(1+\sin t)(1-\sin t)}$$

$$16. \frac{1+\csc \beta}{\cot \beta + \cos \beta} = \sec \beta$$

$$\frac{1 + \frac{1}{\sin B}}{\frac{\cos B}{\sin B} + \cos B} =$$

$$\frac{\sin B + 1}{\sin B}$$

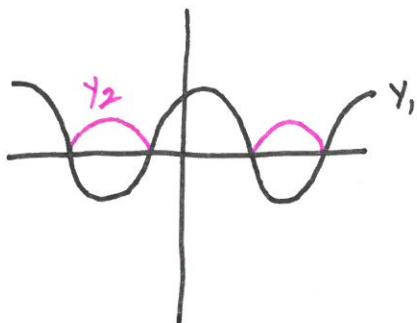
$$\frac{\sin B + 1}{\cos B + \frac{\cos B \sin B}{\sin B}} = \frac{\sin B + 1}{\cos B(1 + \sin B)} = \frac{1}{\cos B} = \sec B$$

$$\sin^2 x = (\sin(x))^2 \text{ or } \sin(x)^2$$

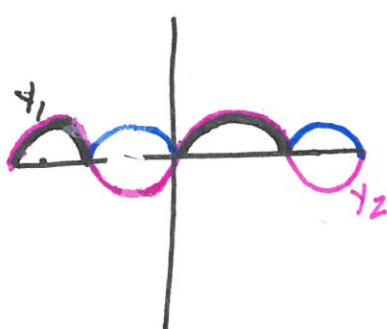
Show that the equation is *not* an identity. (Hint: Graph \odot and show that the left side \neq the right side)

17. $\cos t = \sqrt{1 - \sin^2 t}$

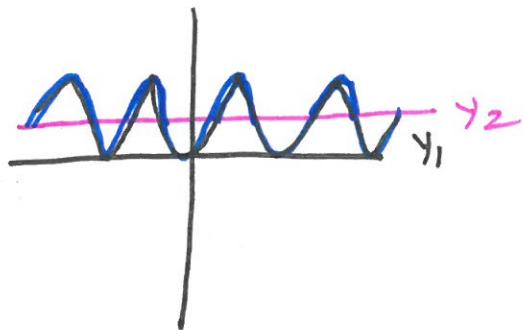
y_1 , y_2



18. $\sqrt{\sin^2 t} = \sin t$



19. $(\sin \theta + \cos \theta)^2 = \sin^2 \theta + \cos^2 \theta$



Make the trigonometric substitution $x = a \sin \theta$ for $-\pi/2 < \theta < \pi/2$ and $a > 0$. Use fundamental identities to simplify the resulting expression.

$$20. (a^2 - x^2)^{3/2} = (\sqrt{a^2 - x^2})^3 = (\sqrt{a^2 - a^2 \sin^2 \theta})^3 = (\sqrt{a^2(1 - \sin^2 \theta)})^3 \\ = (\sqrt{a^2 \cos^2 \theta})^3 = (a \cdot \cos \theta)^3 \\ = \boxed{a^3 \cos^3 \theta}$$

Make the trigonometric substitution $x = a \tan \theta$ for $-\pi/2 < \theta < \pi/2$ and $a > 0$. Simplify the resulting expression.

$$21. \sqrt{a^2 + x^2} = \sqrt{a^2 + a^2 \tan^2 \theta} = \sqrt{a^2(1 + \tan^2 \theta)} = \sqrt{a^2 \cdot \sec^2 \theta} \\ = \boxed{a \cdot \sec \theta}$$

Make the trigonometric substitution $x = a \sec \theta$ for $0 < \theta < \pi/2$ and $a > 0$. Simplify the resulting expression.

$$22. \sqrt{x^2 - a^2} = \sqrt{a^2 \sec^2 \theta - a^2} = \sqrt{a^2(\sec^2 \theta - 1)} = \sqrt{a^2 \tan^2 \theta} \\ = \boxed{a \cdot \tan \theta}$$

Find the simplest expression.

$$23. f(x) = \frac{\sin x - \sin^3 x}{\cos^4 x + \cos^2 x \sin^2 x} = \frac{\sin x (1 - \sin^2 x)}{\cos^2 x (\cos^2 x + \sin^2 x)} = \frac{\sin x (\cos^2 x)}{\cos^2 x \cdot 1} = \boxed{\sin x}$$