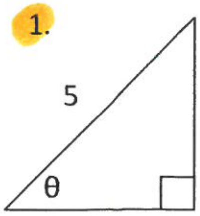
*AAT (IC/HW)-Days 1 & 2 Name Key

Chapter 6: 6-2 Trigonometric Functions of Angles Date: ________ Period: ________

Find the values of the six trigonometric functions for the angle θ.

1.

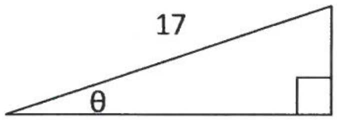



$\sin\theta = \frac{4}{5}$; $\csc\theta = \frac{5}{4}$

$\cos\theta = \frac{3}{5}$; $\sec\theta = \frac{5}{3}$

$\tan\theta = \frac{4}{3}$; $\cot\theta = \frac{3}{4}$

2.

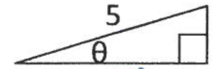



$\sin\theta = \frac{8}{17}$; $\csc\theta = \frac{17}{8}$

$\cos\theta = \frac{15}{17}$; $\sec\theta = \frac{17}{15}$

$\tan\theta = \frac{8}{15}$; $\cot\theta = \frac{15}{8}$

3.



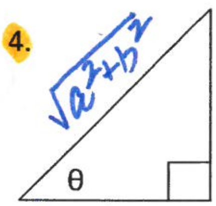

$\sqrt{5^2-2^2} = \sqrt{21}$

$\sin\theta = \frac{2}{5}$; $\csc\theta = \frac{5}{2}$

$\cos\theta = \frac{\sqrt{21}}{5}$; $\sec\theta = \frac{5}{\sqrt{21}}$

$\tan\theta = \frac{2}{\sqrt{21}}$; $\cot\theta = \frac{\sqrt{21}}{2}$

4.

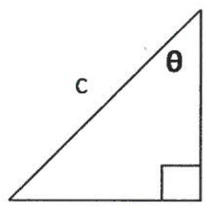



$\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$; $\csc\theta = \frac{\sqrt{a^2+b^2}}{a}$

$\cos\theta = \frac{b}{\sqrt{a^2+b^2}}$; $\sec\theta = \frac{\sqrt{a^2+b^2}}{b}$

$\tan\theta = \frac{a}{b}$; $\cot\theta = \frac{b}{a}$

5.

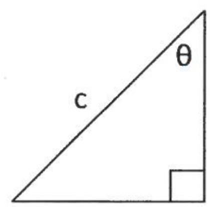



$\sin\theta = \frac{\sqrt{c^2-a^2}}{c}$; $\csc\theta = \frac{c}{\sqrt{c^2-a^2}}$

$\cos\theta = \frac{a}{c}$; $\sec\theta = \frac{c}{a}$

$\tan\theta = \frac{\sqrt{c^2-a^2}}{a}$; $\cot\theta = \frac{a}{\sqrt{c^2-a^2}}$

6.




$\sin\theta = \frac{b}{c}$; $\csc\theta = \frac{c}{b}$

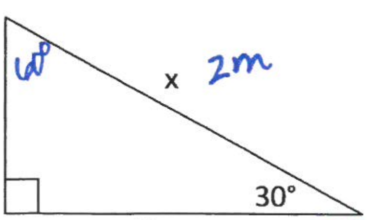
$\cos\theta = \frac{\sqrt{c^2-b^2}}{c}$

$\tan\theta = \frac{b}{\sqrt{c^2-b^2}}$; $\sec\theta = \frac{c}{\sqrt{c^2-b^2}}$

$\cot\theta = \frac{\sqrt{c^2-b^2}}{b}$

Find the exact values of x and y.

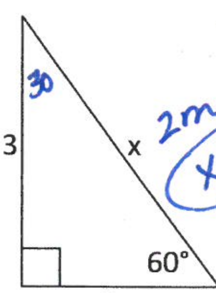
7.




$x = 8$

$y = 4\sqrt{3}$

8.

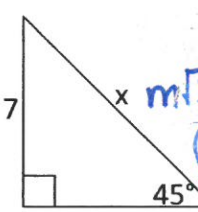



$x = 2\sqrt{3}$

$\frac{3}{\sqrt{3}} = \frac{m\sqrt{3}}{\sqrt{3}}$ $m = \sqrt{3}$

$y = \sqrt{3}$

9.

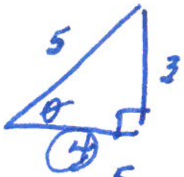



$x = 7\sqrt{2}$

$y = 7$

**Find the exact values of the trigonometric functions for the acute angle θ.**

10. $\sin\theta = \frac{3}{5}$

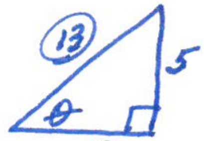



$\sin\theta = \frac{3}{5}$; $\csc\theta = \frac{5}{3}$

$\cos\theta = \frac{4}{5}$; $\sec\theta = \frac{5}{4}$

$\tan\theta = \frac{3}{4}$; $\cot\theta = \frac{4}{3}$

11. $\tan\theta = \frac{5}{12}$

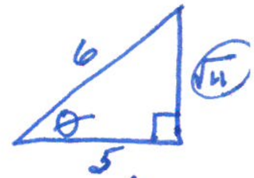



$\sin\theta = \frac{5}{13}$; $\csc\theta = \frac{13}{5}$

$\cos\theta = \frac{12}{13}$; $\sec\theta = \frac{13}{12}$

$\tan\theta = \frac{5}{12}$; $\cot\theta = \frac{12}{5}$

12. $\sec\theta = \frac{6}{5}$

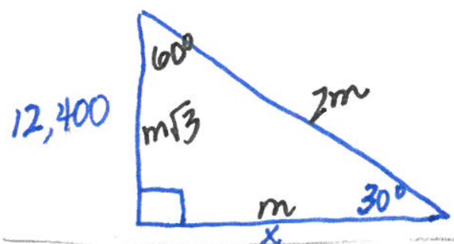



$\sin\theta = \frac{\sqrt{11}}{6}$; $\csc\theta = \frac{6}{\sqrt{11}}$

$\cos\theta = \frac{5}{6}$; $\sec\theta = \frac{6}{5}$

$\tan\theta = \frac{\sqrt{11}}{5}$; $\cot\theta = \frac{5}{\sqrt{11}}$

13. The peak of Mt. Fuji in Japan is approximately 12,400 feet high. A trigonometry student, several miles away, notes that the angle between level ground and the peak is 30°. Estimate the distance from the student to the point on level ground directly beneath the peak.




$\tan 30 = \frac{12,400}{x}$

$x \approx 21,477.4$ ft.

-or-

$x = 12,400\sqrt{3}$

$\approx 21,477.4$ ft

**Approximate to four decimal places.**

14. (a) $\sin 42° \approx 0.6691$

(b) $\cos 77° \approx 0.2250$

(c) $\csc 123° = \frac{1}{\sin 123} \approx 1.1924$ ~~x~~ mode (deg)

(d) $\sec(-190°) = \frac{1}{\cos -190} \approx -1.0154$

(e) $\cot\left(\frac{\pi}{13}\right) = \frac{1}{\tan\frac{\pi}{13}} \approx 4.0572$

(f) $\csc 1.32 = \frac{1}{\sin 1.32} \approx 1.0323$ — mode (rad)

(g) $\cos(-8.54) \approx -0.6335$

(h) $\tan\left(\frac{3\pi}{7}\right) \approx 4.3813$

**Use the Pythagorean identities to write the expression as an integer.**

15. (a) $\tan^2 4\beta - \sec^2 4\beta$

Since $1 + \tan^2 4B = \sec^2 4B$

$\tan^2 4B - \sec^2 4B = -1$

(b) $4\tan^2\beta - 4\sec^2\beta$

$4(\tan^2 B - \sec^2 B) = 4(-1) = -4$

16. (a) $7\sec^2\gamma - 7\tan^2\gamma$

$7(\sec^2 y - \tan^2 y)$

$7(1) = 7$

(b) $7\sec^2\left(\frac{\gamma}{3}\right) - 7\tan^2\left(\frac{\gamma}{3}\right)$

$7\left(\sec^2\frac{y}{3} - \tan^2\frac{y}{3}\right)$

$7(1) = 7$

**Simplify the expression.**

17. $\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} = \frac{\cancel{(\sin\theta + \cos\theta)}(\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta)}{\cancel{(\sin\theta + \cos\theta)}}$

$= 1 - \sin\theta\cos\theta$

18. $\frac{2 - \tan\theta}{2\csc\theta - \sec\theta} = \frac{2 - \frac{\sin\theta}{\cos\theta}}{2\left(\frac{1}{\sin\theta}\right) - \left(\frac{1}{\cos\theta}\right)} = \frac{\frac{2\cos\theta - \sin\theta}{\cos\theta}}{\frac{2\cos\theta - \sin\theta}{\sin\theta \cdot \cos\theta}}$

$\frac{\cancel{2\cos\theta - \sin\theta}}{\cancel{\cos\theta}} \cdot \frac{\sin\theta \cdot \cancel{\cos\theta}}{\cancel{2\cos\theta - \sin\theta}}$

$= \sin\theta$

**Use fundamental identities to write the first expression in terms of the second, for any acute angle θ.**

19. cot θ, sin θ

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta}$

20. sec θ, sin θ

$\sec\theta = \dfrac{1}{\cos\theta}$

$= \dfrac{1}{\sqrt{1-\sin^2\theta}}$

21. sin θ, sec θ

$\sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{1-\dfrac{1}{\sec^2\theta}}$

**Verify the identity by transforming the left-hand side into the right-hand side.**

22. cos θ sec θ = 1

$\cancel{\cos}\theta \cdot \dfrac{1}{\cancel{\cos}\theta} =$

$1 = 1$

23. sin θ sec θ = tan θ

$\sin\theta \cdot \dfrac{1}{\cos\theta} =$

$\dfrac{\sin\theta}{\cos\theta} =$

$\tan\theta = \tan\theta$

24. $\dfrac{\csc\theta}{\sec\theta} = \cot\theta$

$\dfrac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}} =$

$\dfrac{1}{\sin\theta} \cdot \dfrac{\cos\theta}{1} =$

$\dfrac{\cos\theta}{\sin\theta} = \cot\theta = \cot\theta$

25. $(1 + \cos 2\theta)(1 - \cos 2\theta) = \sin^2 2\theta$

$= 1 - \cos^2 2\theta$

$= \sin^2 2\theta = \sin^2 2\theta$

26. $\cos^2\theta(\sec^2\theta - 1) = \sin^2\theta$

$= \cos^2\theta(\tan^2\theta)$

$= \cancel{\cos^2\theta}\left(\dfrac{\sin^2\theta}{\cancel{\cos^2\theta}}\right)$

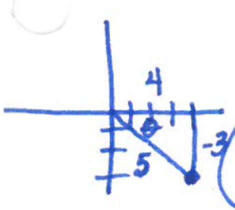
$= \sin^2\theta = \sin^2\theta$

**Find the exact values of the six trigonometric functions of θ if θ is in standard position and P is on the terminal side.**

27. P(4, -3)

$r = \sqrt{4^2 + (-3)^2}$

$r = \sqrt{25} = 5$

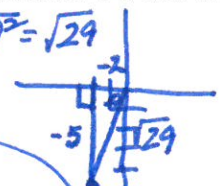



$-3/5,\ 4/5,\ -3/4$

$-5/3,\ 5/4,\ -4/3$

28. P(-2, -5)

$r = \sqrt{(-2)^2 + (-5)^2} = \sqrt{29}$

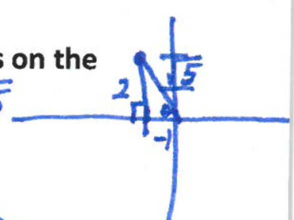



$\dfrac{-5}{\sqrt{29}},\ \dfrac{-2}{\sqrt{29}},\ \dfrac{5}{2}$

$\dfrac{\sqrt{29}}{-5},\ \dfrac{\sqrt{29}}{-2},\ \dfrac{2}{5}$

29. P(-1, 2)

$r = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$

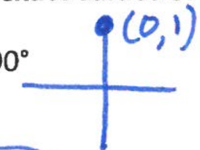



$2/\sqrt{5},\ -1/\sqrt{5},\ -2$

$\sqrt{5}/2,\ \sqrt{5}/-1,\ -\frac{1}{2}$

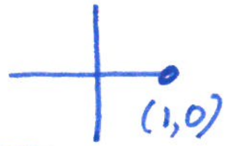
**Find the exact values of the six trigonometric functions of each angle, whenever possible.**

30. (a) 90°



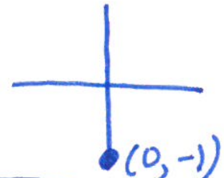

1, 0, u
1, u, 0

(b) 0°



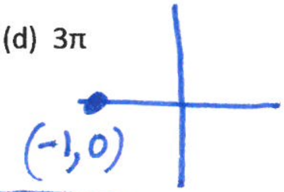

0, 1, 0
u, 1, u

(c) $7\pi/2$




-1, 0, u
-1, u, 0

(d) $3\pi$




0, -1, 0
u, -1, u

**Find the quadrant containing θ if the given conditions are true.**

31. (a) $\cos\theta > 0$ and $\sin\theta < 0$

QI, QIV  QIII, QIV

QIV

(b) $\sin\theta < 0$ and $\cot\theta > 0$

QIII, QIV  QI, QIII

QIII

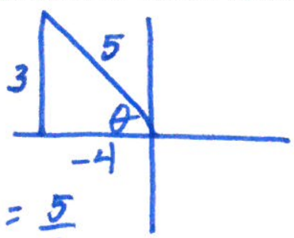
(c) $\csc\theta > 0$ and $\sec\theta < 0$

QI, QII  QII, QIII

QII

**Use fundamental identities to find the values of the trigonometric functions for the given conditions.**

32. $\tan\theta = -\frac{3}{4}$ and $\sin\theta > 0$

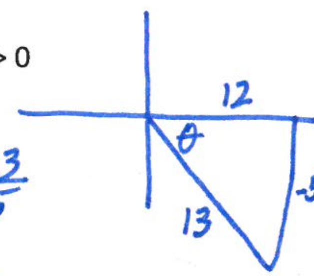



$\sin\theta = \frac{3}{5}$ ; $\csc\theta = \frac{5}{3}$

$\cos\theta = \frac{-4}{5}$ ; $\sec\theta = \frac{-5}{4}$

$\tan\theta = \frac{-3}{4}$ ; $\cot\theta = -\frac{4}{3}$

33. $\sin\theta = -\frac{5}{13}$ and $\sec\theta > 0$




$\sin\theta = \frac{-5}{13}$ ; $\csc\theta = \frac{-13}{5}$

$\cos\theta = \frac{12}{13}$ ; $\sec\theta = \frac{13}{12}$

$\tan\theta = \frac{-5}{12}$ ; $\cot\theta = \frac{-12}{5}$

**Rewrite the expression in nonradical form without using absolute values for the indicated values of θ.**

34. $\sqrt{\sec^2\theta - 1}$; $\pi/2 < \theta < \pi$

$= \sqrt{\tan^2\theta}$

$= |\tan\theta|$

$= -\tan\theta$ since QII

35. $\sqrt{1 + \cot^2\theta}$; $0 < \theta < \pi$

$= \sqrt{\csc^2\theta}$

$= |\csc\theta|$

$= \csc\theta$ since QI $\&$ QII