

If the given angle is in standard position, find two positive coterminal angles and two negative coterminal angles.

1. (a)  $120^\circ$

(b)  $135^\circ$

(c)  $-30^\circ$

(d)  $620^\circ$

$$\begin{array}{llll}
 120^\circ + 1(360^\circ) = 480^\circ & 135^\circ + 1(360^\circ) = 495^\circ & -30^\circ + 1(360^\circ) = 330^\circ & 620^\circ + 1(360^\circ) = 980^\circ \\
 120^\circ + 2(360^\circ) = 840^\circ & 135^\circ + 2(360^\circ) = 855^\circ & -30^\circ + 2(360^\circ) = 690^\circ & 620^\circ - 1(360^\circ) = 260^\circ \\
 120^\circ - 1(360^\circ) = -240^\circ & 135^\circ - 1(360^\circ) = -225^\circ & -30^\circ - 1(360^\circ) = -390^\circ & 620^\circ - 2(360^\circ) = -100^\circ \\
 120^\circ - 2(360^\circ) = -600^\circ & 135^\circ - 2(360^\circ) = -585^\circ & -30^\circ - 2(360^\circ) = -750^\circ & 620^\circ - 3(360^\circ) = -460^\circ
 \end{array}$$

(e)  $\frac{5\pi}{6}$

(f)  $-\frac{\pi}{4}$

(g)  $\frac{2\pi}{3}$

(h)  $-\frac{5\pi}{4}$

$$\begin{array}{llll}
 \frac{5\pi}{6} + 1(2\pi) = \frac{17\pi}{6} & -\frac{\pi}{4} + 1(2\pi) = \frac{7\pi}{4} & \frac{2\pi}{3} + 1(2\pi) = \frac{8\pi}{3} & -\frac{5\pi}{4} + 1(2\pi) = \frac{3\pi}{4} \\
 \frac{5\pi}{6} + 2(2\pi) = \frac{29\pi}{6} & -\frac{\pi}{4} + 2(2\pi) = \frac{15\pi}{4} & \frac{2\pi}{3} + 2(2\pi) = \frac{14\pi}{3} & -\frac{5\pi}{4} + 2(2\pi) = \frac{11\pi}{4} \\
 \frac{5\pi}{6} - 1(2\pi) = -\frac{7\pi}{6} & -\frac{\pi}{4} - 1(2\pi) = -\frac{9\pi}{4} & \frac{2\pi}{3} - 1(2\pi) = -\frac{4\pi}{3} & -\frac{5\pi}{4} - 1(2\pi) = -\frac{13\pi}{4} \\
 \frac{5\pi}{6} - 2(2\pi) = -\frac{19\pi}{6} & -\frac{\pi}{4} - 2(2\pi) = -\frac{17\pi}{4} & \frac{2\pi}{3} - 2(2\pi) = -\frac{10\pi}{3} & -\frac{5\pi}{4} - 2(2\pi) = -\frac{21\pi}{4}
 \end{array}$$

Find the angle that is complementary to  $\theta$ .

2. (a)  $\theta = 5^\circ 17' 34''$

$90^\circ - 5^\circ 17' 34'' = 84^\circ 42' 26''$

2nd angle  
" → above + key  
DMS → 2nd L

(b)  $\theta = 32.5^\circ$

$90^\circ - 32.5^\circ = 57.5^\circ$

Find the angle that is supplementary to  $\theta$ .

3. (a)  $\theta = 48^\circ 51' 37''$

$180^\circ - 48^\circ 51' 37'' = 131^\circ 8' 23''$

(b)  $\theta = 136.42^\circ$

$180^\circ - 136.42^\circ = 43.58^\circ$

Find the exact radian measure of the angle.

4. (a)  $150^\circ$

$$150^\circ \cdot \frac{\pi}{180^\circ} = \frac{5\pi}{6}$$

(b)  $-60^\circ$

$$-60^\circ \cdot \frac{\pi}{180^\circ} = -\frac{\pi}{3}$$

(c)  $225^\circ$

$$225^\circ \cdot \frac{\pi}{180^\circ} = \frac{5\pi}{4}$$

(d)  $450^\circ$

$$450^\circ \cdot \frac{\pi}{180^\circ} = \frac{5\pi}{2}$$

(e)  $72^\circ$

$$72^\circ \cdot \frac{\pi}{180^\circ} = \frac{2\pi}{5}$$

(f)  $100^\circ$

$$100^\circ \cdot \frac{\pi}{180^\circ} = \frac{5\pi}{9}$$

Find the exact degree measure of the angle.

5. (a)  $\frac{2\pi}{3}$

$$\frac{2\pi}{3} \cdot \frac{180}{\pi} = 120^\circ$$

(b)  $\frac{11\pi}{6}$

$$\frac{11\pi}{6} \cdot \frac{180}{\pi} = 330^\circ$$

(c)  $\frac{3\pi}{4}$

$$\frac{3\pi}{4} \cdot \frac{180}{\pi} = 135^\circ$$

(d)  $-\frac{7\pi}{2}$

$$-\frac{7\pi}{2} \cdot \frac{180}{\pi} = -630^\circ$$

(e)  $\frac{\pi}{9}$

$$\frac{\pi}{9} \cdot \frac{180}{\pi} = 20^\circ$$

(f)  $7\pi$

$$7\pi \cdot \frac{180}{\pi} = 1260^\circ$$

Express  $\theta$  in terms of degrees, minutes, and seconds, to the nearest second.

6.  $\theta = 2$

$$2 \cdot \frac{180}{\pi} \approx 114.59156^\circ \triangleright \text{DMS}$$

$$\approx 114^\circ 35' 30''$$

7.  $\theta = 5$

$$5 \cdot \frac{180}{\pi} \approx 286.4787^\circ$$

$$\approx 286^\circ 28' 44''$$

Express the angle as a decimal, to the nearest ten-thousandth of a degree.

8.  $37^\circ 41'$

$$\approx 37.6833^\circ$$

9.  $115^\circ 26' 27''$

$$\approx 115.4408^\circ$$

Express the angle in terms of degrees, minutes, and seconds, to the nearest second.

10.  $63.169^\circ \triangleright \text{DMS}$

$$\approx 63^\circ 10' 8''$$

11.  $310.6215^\circ \triangleright \text{DMS}$

$$\approx 310^\circ 37' 17''$$

If a circular arc of the given length  $s$  subtends the central angle  $\theta$  on a circle, find the radius of the circle.

12.  $s = 10$  cm,  $\theta = 4$

$$s = r\theta$$

$$\frac{10}{4} = \frac{r \cdot 4}{4}$$

$$r = 2.5 \text{ cm}$$

13.  $s = 3$  km,  $\theta = 20^\circ$

$$s = r\theta$$

$$3 = r \cdot \left( \frac{20 \cdot \pi}{180} \right)$$

$$3 = \frac{\pi}{9} r$$

$$r = \frac{27}{\pi} \approx 8.59 \text{ km}$$

Find (a) the length of the arc and (b) the area of the circle sector.

14. central angle  $\theta = 45^\circ$ ,  $r = 8$  cm.

(a)  $s = r \cdot \theta$

$$= 8 \left( 45 \cdot \frac{\pi}{180} \right)$$

$$s = 8 \left( \frac{\pi}{4} \right) = 2\pi \approx 6.28$$

(b)  $A = \frac{1}{2} r^2 \theta$  -or-  $A = \frac{\theta}{360} \cdot \pi r^2$

$$= \frac{1}{2} (8)^2 \left( \frac{\pi}{4} \right)$$

$$A = 8\pi \approx 25.13 \text{ cm}^2$$

$$= \frac{45}{360} \cdot \pi (8)^2$$

$$= 8\pi \approx 25.13 \text{ cm}^2$$

Find (a) the radian and degree measures of the central angle  $\theta$  subtended by the given arc of length  $s$  on a circle of radius  $r$  and (b) find the area of the sector determined by  $\theta$ .

15.  $s = 7$  cm,  $r = 4$  cm

(a)  $s = r \cdot \theta$

$$\frac{7}{4} = \frac{4 \cdot \theta}{4}$$

$$\theta = \frac{7}{4} = 1.75 \text{ radians}$$

$$\frac{7}{4} \cdot \frac{180^\circ}{\pi} \approx 100.27^\circ$$

(b)  $A = \frac{1}{2} r^2 \theta$  -or-  $A = \frac{\theta}{360} \cdot \pi r^2$

$$= \frac{1}{2} (4^2) \left( \frac{7}{4} \right)$$

$$= 14 \text{ cm}^2$$

$$= \frac{100.27}{360} \cdot \pi (4)^2 \approx 14 \text{ cm}^2$$

16. A wheel of the given radius 5 in. is rotating at the rate of 40 rpm. Find the angular speed (in radians per minute).

$$\left( 40 \frac{\text{rev}}{\text{min}} \right) \left( 2\pi \frac{\text{rad}}{\text{rev}} \right) = 80\pi \frac{\text{rad}}{\text{min}}$$

$$A = (\text{RPM}) \times 2\pi$$

17. A typical tire for a compact car is 22 inches in diameter. If the car is traveling at a speed of 60 mi/hr, find the number of revolutions the tire makes per minute.

$$\frac{60 \text{ mi}}{\text{hr}} = \frac{60 \text{ mil}}{\text{hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{5280 \text{ ft}}{\text{mi}} \cdot \frac{12 \text{ in}}{\text{ft}} = \frac{63,360 \text{ in.}}{\text{min}}$$

### Dimensional Analysis

18. A vendor sells two sizes of pizza by the slice. The small slice is  $\frac{1}{6}$  of a circular 18-inch-diameter pizza, and it sells for \$2.00. The large slice is  $\frac{1}{8}$  of a circular 26-inch-diameter pizza, and it sells for \$3.00. Which slice provides more pizza per dollar?

$$A_{\text{small}} = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} \left( \frac{1}{2} (18) \right)^2 \left( \frac{2\pi}{6} \right)$$

$$= \frac{27\pi}{2} \therefore \text{Cost}_{\text{small}} = \frac{27\pi}{2} \div 2 \approx 21.21 \text{ in}^2/\$$$

provides slightly more pizza per \$

$$A_{\text{large}} = \frac{1}{2} \left( \frac{1}{2} \cdot 26 \right)^2 \cdot \left( \frac{2\pi}{8} \right) = \frac{169\pi}{8} \therefore \text{Cost}_{\text{large}} =$$

$$\frac{169\pi}{8} \div 3 \approx 22.12 \text{ in}^2/\$$$