*AAT (IC/HW)-Day 1

Name: Key

Chapter 5: 5-5 Exponential and Logarithmic Equations

Date: ____________ Period: ____________

Find the exact solution and a two-decimal-place approximation for it by using (a) logarithms & (b) the change of base formula.

1. $5^x = 8$

(a) $5^x = 8$

$\log 5^x = \log 8$

$x \log 5 = \log 8$

$x = \frac{\log 8}{\log 5} \approx 1.29$

(b) $x = \log_5 8$

$x = \frac{\log 8}{\log 5}$

≈ 1.29

2. $3^{4-x} = 5$

(a) $\log 3^{4-x} = \log 5$

$(4-x)\log 3 = \log 5$

$4 - x = \frac{\log 5}{\log 3}$

$x = 4 - \frac{\log 5}{\log 3} \approx 2.54$

(b) $3^{4-x} = 5$

$4 - x = \log_3 5$

$x = 4 - \frac{\log 5}{\log 3}$

≈ 2.54

3. $(\frac{1}{3})^x = 100$

(a) $\log (\frac{1}{3})^x = \log 100$

$x \log \frac{1}{3} = 2$

$x = \frac{2}{\log \frac{1}{3}}$

≈ -4.19

(b) $x = \log_{\frac{1}{3}} 100$

$x = \frac{\log 100}{\log \frac{1}{3}}$

≈ -4.19

Estimate using the change of base formula.

4. $\log_5 6$

$\frac{\log 6}{\log 5} \approx 1.1133$

5. $\log_9 0.2$

$\frac{\log .2}{\log 9} \approx -.7325$

Evaluate using the change of base formula (without a calculator.)

6. $\frac{\log_5 16}{\log_5 4}$

$= \frac{\frac{\log 16}{\log 5}}{\frac{\log 4}{\log 5}} = \frac{\log 16}{\log 4} = \log_4 16 = \log_4 4^2 = 2$

7. $\frac{\log_7 243}{\log_7 3}$

$= \log_3 243$

$= \log_3 3^5 = 5$

Find (a) the exact solution, using common logarithms, and (b) a two-decimal approximation of each solution, when appropriate.

8. $3^{x+4} = 2^{1-3x}$

$= (x+4)\log 3 = (1-3x)\log 2$

$x \log 3 + 4\log 3 = \log 2 - 3x \log 2$

$x \log 3 + 3x \log 2 = \log 2 - 4\log 3$

$\frac{x(\log 3 + 3\log 2)}{(\log 3 + 3\log 2)} = \frac{\log 2 - 4\log 3}{(\log 3 + 3\log 2)}$

$x \approx -1.16$

9. $2^{2x-3} = 5^{x-2}$

$(2x-3)\log 2 = (x-2)\log 5$

$2x\log 2 - 3\log 2 = x\log 5 - 2\log 5$

$2x\log 2 - x\log 5 = 3\log 2 - 2\log 5$

$\frac{x(2\log 2 - \log 5)}{(2\log 2 - \log 5)} = \frac{3\log 2 - 2\log 5}{(2\log 2 - \log 5)}$

$x \approx 5.11$

10. $2^{-x} = 8$

$2^{-x} = 2^3$

$-x = 3$

$x = -3$

11. log x = 1 - log (x - 3)

$\log x + \log(x-3) = 1$

$\log x^2 - 3x = 1$

$10^1 = x^2 - 3x$

$0 = x^2 - 3x - 10$

$0 = (x-5)(x+2)$

$x = 5$; $\cancel{x = -2}$

12. log (x² + 4) - log (x + 2) = 2 + log (x - 2)

$\log \frac{x^2+4}{x+2} - \log(x-2) = 2$

$\log\left(\frac{x^2+4}{x^2-4}\right) = 2$

$10^2 = \left(\frac{x^2+4}{x^2-4}\right)$

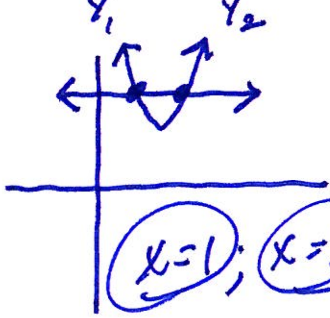
$x^2 + 4 = 100x^2 - 400$;

$\frac{404}{99} = \frac{99x^2}{99}$

$x = 2.02$

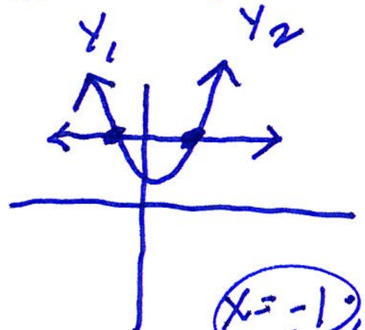
$x = \cancel{-2.02}$

13. $\underbrace{5^x + 125(5^{-x})}_{Y_1} = \underbrace{30}_{Y_2}$



$x = 1$; $x = 2$

14. $\underbrace{3(3^x) + 9(3^{-x})}_{Y_1} = \underbrace{28}_{Y_2}$



$x = -1$; $x = 2$

Solve the equation without using a calculator.

15. log(x²) = (log x)²

$2\log x = (\log x)^2$

$0 = (\log x)^2 - 2\log x$

$0 = \log x(\log x - 2)$

$\log x = 0$ $\quad$ $\log x = 2$

$10^0 = x$ $\quad$ $10^2 = x$

$x = 1$ $\quad$ $x = 100$

16. log (log x) = 2

$10^2 = \log x$

$100 = \log x$

$10^{100} = x$

17. $x^{\sqrt{\log x}} = 10^8$

$\log\left(x^{\sqrt{\log x}}\right) = \log 10^8$

$\sqrt{\log x}\,(\log x) = 8 \log 10$

$(\log x)^{\frac{3}{2}} = 8$

$\log x = 8^{\frac{2}{3}}$

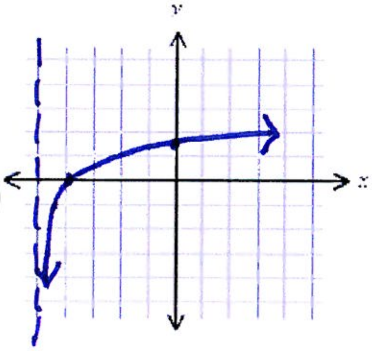
$\log x = 4$

$10^4 = x$

$10{,}000 = x$

Sketch the graph of f, and use the change of base formula to approximate the y-intercept.

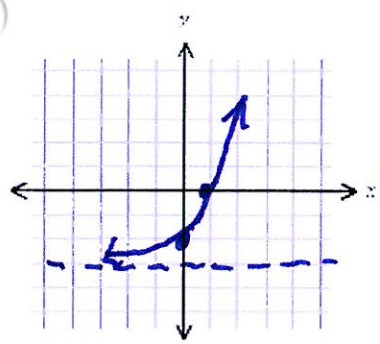
18. f(x) = log₃ (x + 5)



y-int; $\log_3 5 = \frac{\log 5}{\log 3} \approx 1.4650$

Sketch the graph of f, and use the change of base formula to approximate the x-intercept.

19. $f(x) = 4^x - 3$



x-int;

$y = 4^x - 3$

$0 = 4^x - 3$

$3 = 4^x$

$x = \log_4 3$; $\frac{\log 3}{\log 4} \approx .7925$

20. Use the compound interest formula to determine how long it will take for a sum of money to double if it is invested at a rate of 6% per year compounded monthly.

$A = 2P$

$r = 0.06$

$n = 12$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$2P = P\left(1 + \frac{.06}{12}\right)^{12t}$

$2 = (1.005)^{12t}$

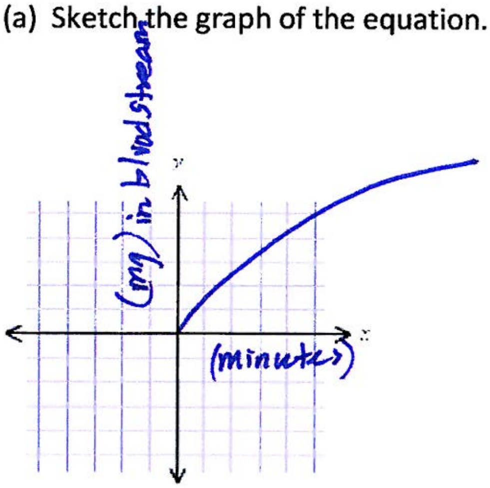
$\ln 2 = 12t \ln 1.005$

$t = \frac{\ln 2}{12 \ln(1.005)} \approx 11.58$ yr or 11 yr. 7 mon

21. If a 100-milligram tablet of an asthma drug is taken orally and if none of the drug is present in the body when the tablet is first taken, the total amount A in the bloodstream after t minutes is predicted to be

$$A = 100[1 - (0.9)^t] \text{ for } 0 \le t \le 10.$$

(a) Sketch the graph of the equation.



(b) Determine the number of minutes needed for 50 milligrams of the drug to have entered the bloodstream.

$A = 50$

$50 = 100(1 - .9^t)$

$.5 = 1 - .9^t$

$\frac{-.5}{-1} = \frac{-.9^t}{-1}$

$.5 = .9^t$

$t = \log_{.9}(.5) = \frac{\log .5}{\log .9} \approx 6.58$ min.