***AAT (IC/HW)-Day 1**
Chapter 5: 5-4 Properties of Logarithms

Name Key
Date: _____ Period: _____

Express in terms of logarithms of x, y, z, or w.

1. (a) $\log_4(xz)$ $= \log_4 x + \log_4 z$

(b) $\log_4(y/x)$ $= \log_4 y - \log_4 x$

(c) $\log_4 \sqrt[3]{z}$ $= \log_4 z^{\frac{1}{3}} = \frac{1}{3}\log_4 z$

2. $\log_a \frac{x^3 w}{y^2 z^4} = \log_a x^3 w - \log_a y^2 z^4$
$= \log_a x^3 + \log_a w - (\log_a y^2 + \log_a z^4)$
$= 3\log_a x + \log_a w - 2\log_a y - 4\log_a z$

3. $\log \frac{\sqrt{y}}{x^4\sqrt[3]{z}} = \log y^{\frac{1}{2}} - \log x^4 - \log z^{\frac{1}{3}}$
$= \frac{1}{2}\log y - 4\log x - \frac{1}{3}\log z$

4. $\ln \sqrt[4]{\frac{x^7}{y^5 z}} = \ln x^{\frac{7}{4}} - \ln y^{\frac{5}{4}} z^{\frac{1}{4}}$
$= \ln x^{\frac{7}{4}} - \ln y^{\frac{5}{4}} - \ln z^{\frac{1}{4}}$
$= \frac{7}{4}\ln x - \frac{5}{4}\ln y - \frac{1}{4}\ln z$

Write the expression as one logarithm.

5. (a) $\log_3 x + \log_3(5y)$
$\log_3(x \cdot 5y)$
$\log_3 5xy$

(b) $\log_3(2z) - \log_3 x$
$= \log_3 \frac{2z}{x}$

(c) $5\log_3 y$
$\log_3 y^5$

6. $2\log_a x + \frac{1}{3}\log_a(x-2) - 5\log_a(2x+3) =$
$\log_a x^2 + \log_a (x-2)^{\frac{1}{3}} - \log_a (2x+3)^5$
$\log_a \frac{x^2 \sqrt[3]{x-2}}{(2x+3)^5}$

7. $\log(x^3y^2) - 2\log x\sqrt[3]{y} - 3\log(\frac{x}{y})$
$\log \frac{x^3 y^2}{x^2 y^{\frac{2}{3}} \cdot \frac{x^3}{y^3}} = \log \frac{x^3 y^2}{x^5 y^{\frac{2}{3}-3}} = \log \frac{x^3 y^2}{x^5 y^{-\frac{7}{3}}}$
$= \log \frac{y^{13/3}}{x^2}$

8. $\ln y^3 + \frac{1}{3}\ln(x^3y^6) - 5\ln y$
$= \ln y^3 + \ln xy^2 - \ln y^5$
$= \ln \frac{xy^5}{y^5} = \ln x$

9. $2\ln x - 4\ln(\frac{1}{y}) - 3\ln(xy)$
$= \ln x^2 - \ln(\frac{1}{y^4}) - \ln x^3 y^3$
$= \ln \frac{x^2}{x^3/y} = \ln(\frac{y}{x})$

Solve the equation.

10. $\log_6(2x - 3) = \log_6 12 - \log_6 3$

$\log_6(2x-3) = \log_6\left(\frac{12}{3}\right)$

$2x-3 = 4$

$2x = 7$

$x = \frac{7}{2}$

11. $2\log_3 x = 3\log_3 5$

$\log_3 x^2 = \log_3 5^3$

$x^2 = 125$

$x = 5\sqrt{5}$

12. $\log x - \log(x+1) = 3\log 4$

$\log \frac{x}{x+1} = \log 4^3$

$\frac{x}{x+1} = 64$

$x = 64x + 64$

$-63x = 64$; $x = \frac{-64}{63}$; $\emptyset$

13. $\ln(-4 - x) + \ln 3 = \ln(2 - x)$

$\ln 3(-4-x) = \ln(2-x)$

$\ln(-12-3x) = \ln(2-x)$

$-12-3x = 2-x$

$-14 = 2x$

$x = -7$

14. $\log_2(x+7) + \log_2 x = 3$

$\log_2 x(x+7) = 3$

$\log_2 x^2 + 7x = 3$

$2^3 = x^2 + 7x$

$0 = x^2 + 7x - 8$

$0 = (x+8)(x-1)$

$x = \cancel{-8}$; $x = 1$

15. $\log_3(x + 3) + \log_3(x+5) = 1$

$\log_3(x+3)(x+5) = 1$

$3^1 = x^2 + 8x + 15$

$0 = x^2 + 8x + 12$

$0 = (x+6)(x+2)$

$x = \cancel{-6}$; $x = -2$

16. $\log_3(x - 2) + \log_3(x - 4) = 2$

$\log_3(x^2 - 6x + 8) = 2$

$3^2 = x^2 - 6x + 8$

$0 = x^2 - 6x - 1$

$x = \frac{6 \pm \sqrt{40}}{2} = \frac{6 \pm 2\sqrt{10}}{2}$

$x = 3 + \sqrt{10}$; $x = \cancel{3 - \sqrt{10}}$

17. $\log(x + 3) = 1 - \log(x - 2)$

$\log(x+3) + \log(x-2) = 1$

$\log(x^2 + x - 6) = 1$

$10^1 = x^2 + x - 6$

$0 = x^2 + x - 16$

$x = 3.53$; $x = \cancel{-4.53}$

18. When the volume control on a stereo system is increased, the voltage across a loudspeaker changes from V1 to V2, and the decibel increase in gain is given by $db = 20 \log \frac{V_2}{V_1}$. Find the decibel increase if the voltage changes from 2 volts to 4.5 volts. $V_1 = 2$; $V_2 = 4.5$

$20 \log \frac{4.5}{2} \approx 7.04$ or $+7$

19. Pareto's law for capitalist countries states that the relationship between annual income x and the number y of individuals whose income exceeds x is $\log y = \log b - k \log x$, where b and k are positive constants. Solve this equation for y.

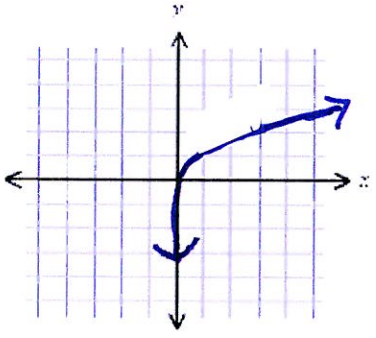
$\log y = \log b - \log x^k$

$\log y = \log \frac{b}{x^k}$

$y = \frac{b}{x^k}$

Sketch the graph of f.

20. $f(x) = \log_3 (3x)$



21. $f(x) = 3 \log_3 x$