

Chapter 4: 4-4 Complex & Rational Zeros of Polynomials

A polynomial $f(x)$ with real coefficients and leading coefficient 1 has the given zero(s) and degree. Express $f(x)$ as a product of linear and quadratic polynomials with real coefficients that are irreducible over \mathbb{R} .

1. $3+2i$; degree 2

$$(x - (3+2i))(x - (3-2i))$$

$$= (x^2 - 6x + 13)$$

2. $2, -2-5i$; degree 3

$$(x-2)(x - (-2-5i))(x - (-2+5i))$$

$$= (x-2)(x^2 + 4x + 29)$$

3. $-1, 0, 3+i$; degree 4

$$(x+1)(x)(x - (3+i))(x - (3-i))$$

$$= x(x+1)(x^2 - 6x + 10)$$

4. $4+3i, -2+i$; degree 4

$$(x - (4+3i))(x - (4-3i))(x - (-2+i))(x - (-2-i))$$

$$= (x^2 - 8x + 25)(x^2 + 4x + 5)$$

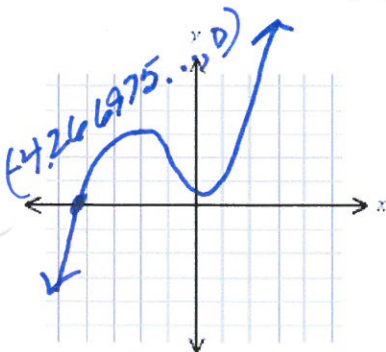
5. $0, -2i, 1-i$; degree 5

$$x(x+2i)(x-2i)(x-(1-i))(x-(1+i))$$

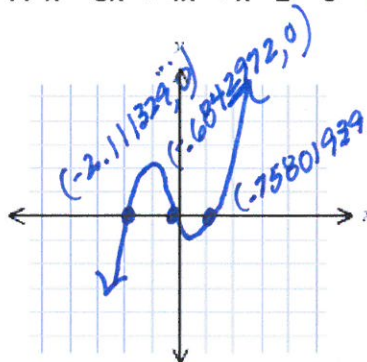
$$= x(x^2+4)(x^2-2x+2)$$

Show that the equation has no rational root.

6. $x^3 + 3x^2 - 4x + 6 = 0$ Possible Rat. Roots $\pm 1, \pm 2, \pm 3, \pm 6$

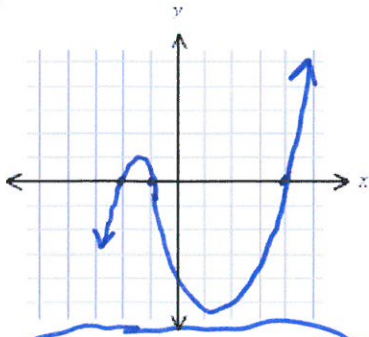


7. $x^5 - 3x^3 + 4x^2 + x - 2 = 0$ Possible Rat. Roots $\pm 1, \pm 2$



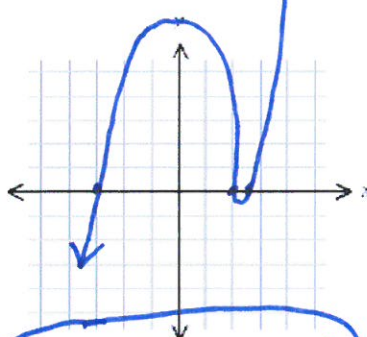
Find all solutions of the equation.

8. $x^3 - x^2 - 10x - 8 = 0$



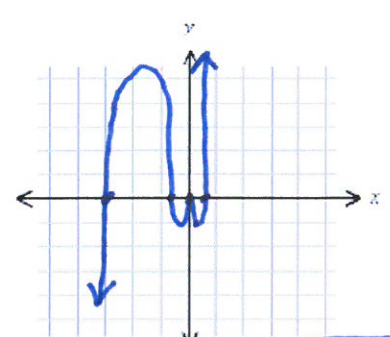
Zeros: $(-2, -1, 4)$

9. $2x^3 - 3x^2 - 17x + 30 = 0$



Zeros: $(-3, 2, 2.5)$

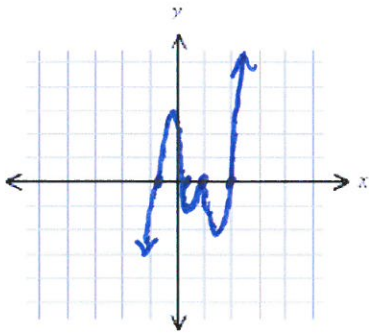
10. $6x^5 + 19x^4 + x^3 - 6x^2 = 0$



Zeros: $-3, -\frac{2}{3}, 0 \text{ (mult. 2)}, \frac{1}{2}$

Find a factored form with integer coefficients of the polynomial.

11. $f(x) = 6x^5 - 23x^4 + 24x^3 + x^2 - 12x + 4 = 0$

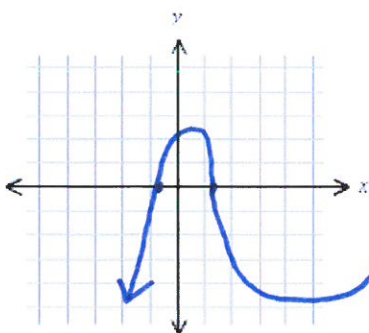


Zeros: $-\frac{2}{3}, \frac{1}{2}, 1 \text{ (mult. 2)}, 2$

$$f(x) = 6\left(x + \frac{2}{3}\right)\left(x - \frac{1}{2}\right)(x-1)^2(x-2)$$

$$f(x) = 6(3x+2)(2x-1)(x-1)^2(x-2)$$

12. The polynomial function $f(x) = 2x^3 - 25.4x^2 + 3.02x + 24.75$ has only real zeros. Use the graph of f to factor it.



$$f(x) = 2(x + 0.9)(x - 1.1)(x - 12.5)$$

Zeros: $-0.9, 1.1, 12.5$

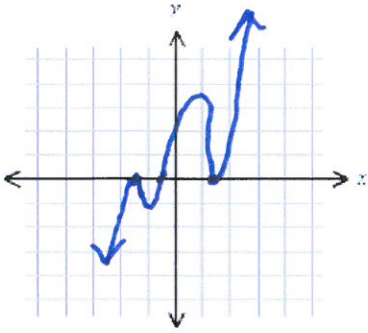
13. Does there exist a polynomial of degree 3 with real coefficients that has zeros 1, -1, and i ? Justify.

No; if i is a root, then $-i$ is also a root

$\therefore \therefore$ would be of degree greater than 3.

Use a graph to determine the number of non real solutions of the equation.

14. $x^5 + 1.1x^4 - 3.21x^3 - 2.835x^2 + 2.7x + .62 = -1$

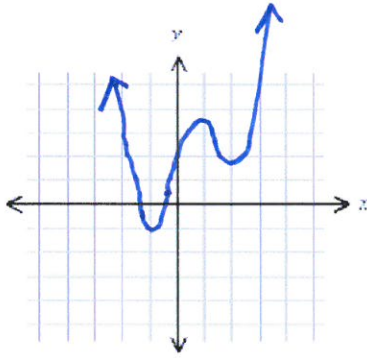


$$y = x^5 + 1.1x^4 - 3.21x^3 - 2.835x^2 + 2.7x + 1.62$$

Zeros: -1.5 (mult. 2), 1.2 (mult. 2), $-.5$ (mult. 1)

No nonreal solutions

15. $x^4 - .4x^3 - 2.6x^2 + 1.1x + 3.5 = 2$



$$y = x^4 - .4x^3 - 2.6x^2 + 1.1x + 1.5$$

Zeros: $-.6$ (mult. 1), -1.4 (mult. 1)

Two nonreal solutions