

Find a polynomial $f(x)$ of degree 3 that has the indicated zeros and satisfies the given condition.

1. -1, 2, 3; $f(-2) = 80$

$$f(x) = a(x+1)(x-2)(x-3)$$

$$f(-2) = a(-1)(-4)(-5) = 80$$

$$\frac{-20a}{-20} = \frac{80}{-20}$$

$$a = -4$$

$$f(x) = -4(x+1)(x-2)(x-3)$$

$$f(x) = -4x^3 + 16x^2 - 4x - 24$$

2. -4, 3, 0; $f(2) = -36$

$$f(x) = a(x+4)(x-3)(x)$$

$$f(2) = a(6)(-1)(2) = -36$$

$$= \frac{-12a}{-12} = \frac{-36}{-12}$$

$$a = 3$$

$$f(x) = 3(x+4)(x-3)(x)$$

$$f(x) = 3x^3 + 3x^2 - 36x$$

3. -2i, 2i, 3; $f(1) = 20$

$$f(x) = a(x+2i)(x-2i)(x-3)$$

$$f(1) = a(1+2i)(1-2i)(-2) = 20$$

$$= \frac{-10a}{-10} = \frac{20}{-10}$$

$$a = -2$$

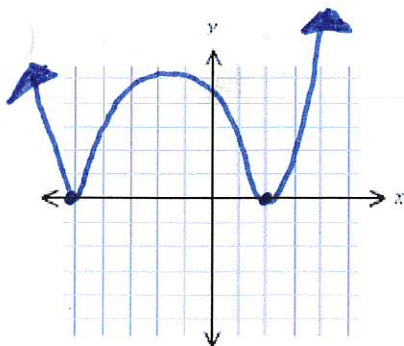
$$f(x) = -2(x+2i)(x-2i)(x-3)$$

$$f(x) = -2x^3 + 6x^2 - 8x + 24$$

4. Find a polynomial $f(x)$ of degree 4 with leading coefficient 1 such that both -5 and 2 are zeros of multiplicity 2, and sketch the graph of f .

$$f(x) = a(x+5)^2(x-2)^2$$

$$f(x) = 1(x^4 + 6x^3 - 11x^2 - 60x + 100)$$



5. Find a polynomial $f(x)$ of degree 6 such that 0 and 3 are both zeros of multiplicity 3 and $f(2) = -24$. Sketch the graph of f .

$$f(x) = a(x)^3(x-3)^3$$

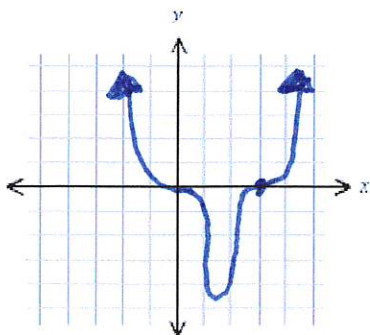
$$f(2) = a(8)(-1) = -24$$

$$\frac{-8a}{-8} = \frac{-24}{-8}$$

$$a = 3$$

$$f(x) = 3(x)^3(x-3)^3$$

$$f(x) = 3x^6 - 27x^5 + 81x^4 - 81x^3$$



6. Find the third-degree polynomial function in factored form with y-intercept (0, 3.5) and x-intercepts (1.5, 0), (-1, 0), & (3, 0).

$$f(x) = a(x+1)(x-1.5)(x-3)$$

$$f(0) = a(1)(-1.5)(-3) = 3.5$$

$$\frac{4.5a}{4.5} = \frac{3.5}{4.5}$$

$$a = \frac{7}{9}$$

$$f(x) = \frac{7}{9}(x+1)(x-1.5)(x-3)$$

Find the zeros of $f(x)$, and state the multiplicity of each zero.

7. $f(x) = x^2(3x+2)(2x-5)^3$

$$-\frac{2}{3} (\text{mult. } 1); 0 (\text{mult. } 2); \frac{5}{2} (\text{mult. } 3)$$

8. $f(x) = 4x^5 + 12x^4 + 9x^3$

$$f(x) = x^3(2x+3)^2$$

$$-\frac{3}{2} (\text{mult. } 2); 0 (\text{mult. } 3)$$

9. $f(x) = (x^2 + x - 12)^3(x^2 - 9)^2$

$$(x+4)^3(x-3)^3(x-3)^2(x+3)^2$$

$$-4 (\text{mult. } 3); 3 (\text{mult. } 5); -3 (\text{mult. } 2)$$

10. $f(x) = x^4 + 7x^2 - 144$

$$f(x) = (x^2+16)(x+3)(x-3)$$

$$\pm 4i (\text{mult. } 1); \pm 3 (\text{mult. } 1)$$

Show that the number is a zero of $f(x)$ of the given multiplicity, and express $f(x)$ as a product of linear factors.

11. $f(x) = x^4 + 7x^3 + 13x^2 - 3x - 18$;

-3(multiplicity 2)

$$\begin{array}{r|rrrrr} -3 & 1 & 7 & 13 & -3 & -18 \\ & & -3 & -12 & -3 & 18 \\ \hline & 1 & 4 & 1 & -6 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 4 & 1 & -6 & 0 \\ & & -3 & -3 & 6 & \\ \hline & 1 & 1 & -2 & 0 & \end{array}$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1)(x+3)^2$$

12. $f(x) = x^6 - 4x^5 + 5x^4 - 5x^2 + 4x - 1$;

1(multiplicity 5)

$$\begin{array}{r|rrrrrrr} 1 & 1 & -4 & 5 & 0 & -5 & 4 & -1 \\ & & 1 & -3 & 2 & 2 & -3 & 1 \\ \hline & 1 & -3 & 2 & 2 & -3 & 1 & 0 \end{array}$$

$$\begin{array}{r|rrrrrr} 1 & 1 & -3 & 2 & 2 & -3 & 1 & 0 \\ & & 1 & -2 & 0 & 2 & -1 & \\ \hline & 1 & -2 & 0 & 2 & -1 & 0 & \end{array}$$

$$\begin{array}{r|rrrrrr} 1 & 1 & -2 & 0 & 2 & -1 & 0 & \\ & & 1 & -1 & -1 & 1 & & \\ \hline & 1 & -1 & -1 & 1 & 0 & & \end{array}$$

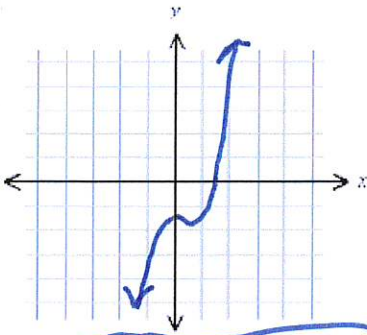
$$\begin{array}{r|rrrrrr} 1 & 1 & -1 & -1 & 1 & 0 & & \\ & & 1 & 0 & -1 & & & \\ \hline & 1 & 0 & -1 & 0 & & & \end{array}$$

$$\begin{array}{r|rrrrrr} 1 & 1 & 0 & -1 & 0 & & & \\ & & 1 & 1 & & & & \\ \hline & 1 & 1 & 0 & & & & \end{array}$$

$$(x-1)^5(x+1)$$

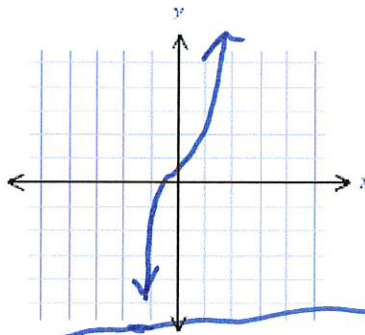
Graph to determine the number of positive, negative, and nonreal complex solutions of the equation.

13. $4x^3 - 6x^2 + x - 3 = 0$



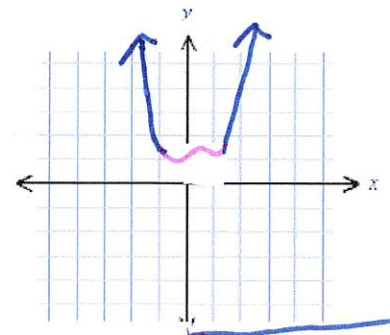
1 pos; 2 nonreal

14. $4x^3 + 2x^2 + 1 = 0$



1 neg; 2 nonreal

15. $3x^4 + 2x^3 - 4x + 2 = 0$

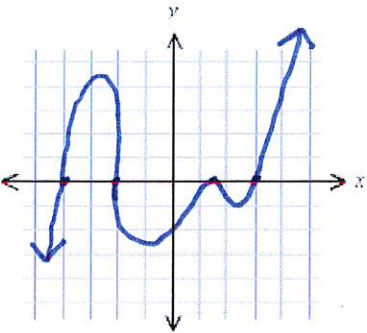


4 nonreal

The polynomial function f has only real zeros. Use the graph of f to factor it.

16. $f(x) = x^5 - 16.75x^3 + 12.75x^2 + 49.5x - 54$

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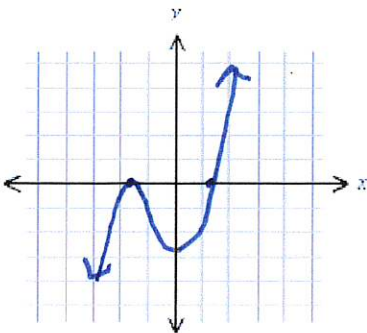


Zeros @ -4, -2, 1.5, 3
(d.r.)

$f(x) = 1(x+4)(x+2)(x-1.5)^2(x-3)$

Graph f , estimate all real zeros, and determine the multiplicity of each zero.

17. $f(x) = x^3 + 1.3x^2 - 1.2x - 1.584$



Zeros @ -1.2 ; 1.1

$-1.2(\text{mult. } 2); 1.1(\text{mult. } 1)$