*AAT

Name: Key

**Chapter 4: 4-2 Properties of Division (IC/HW)-Days 1-2**

Date: ____________ Period: ____________

**Find the quotient and remainder if f(x) is divided by p(x).**

1. $f(x) = 2x^4 - x^3 - 3x^2 + 7x - 12$; $p(x) = x^2 - 3$

$$
\begin{array}{r}
2x^2 - x + 3 \\
x^2+0x-3 \overline{\big)\, 2x^4 - x^3 - 3x^2 + 7x - 12} \\
-\ (2x^4 + 0x^3 - 6x^2) \\
\hline
-x^3 + 3x^2 + 7x \\
-\ (-x^3 - 0x^2 + 3x) \\
\hline
3x^2 + 4x - 12 \\
-\ (3x^2 + 0x - 9) \\
\hline
4x - 3
\end{array}
$$

Q: $2x^2 - x + 3$

R: $4x - 3$

2. $f(x) = 3x^4 + 2x^3 - x^2 - x - 6$; $p(x) = x^2 + 1$

$$
\begin{array}{r}
3x^2 + 2x - 4 \\
x^2+0x+1 \overline{\big)\, 3x^4 + 2x^3 - x^2 - x - 6} \\
-\ (3x^4 + 0x^3 + 3x^2) \\
\hline
2x^3 - 4x^2 - x \\
-\ (2x^3 + 0x^2 + 2x) \\
\hline
-4x^2 - 3x - 6 \\
-\ (-4x^2 - 0x - 4) \\
\hline
-3x - 2
\end{array}
$$

Q: $3x^2 + 2x - 4$

R: $-3x - 2$

3. $f(x) = 3x^3 + 2x - 4$; $p(x) = 2x^2 + 1$

$$
\begin{array}{r}
\tfrac{3}{2}x \\
2x^2+0x+1 \overline{\big)\, 3x^3 + 0x^2 + 2x - 4} \\
-\ (3x^3 + 0x^2 + \tfrac{3}{2}x) \\
\hline
\tfrac{1}{2}x - 4
\end{array}
$$

Q: $\frac{3}{2}x$

R: $\frac{1}{2}x - 4$

4. $f(x) = 3x^5 - 4x^3 + x + 5$; $p(x) = x^3 - 2x + 7$

$$
\begin{array}{r}
3x^2 + 2 \\
x^3+0x^2-2x+7 \overline{\big)\, 3x^5 + 0x^4 - 4x^3 + 0x^2 + x + 5} \\
-\ (3x^5 + 0x^4 - 6x^3 + 21x^2) \\
\hline
2x^3 - 21x^2 + x + 5 \\
-\ (2x^3 + 0x^2 - 4x + 14) \\
\hline
-21x^2 + 5x - 9
\end{array}
$$

Q: $3x^2 + 2$

R: $-21x^2 + 5x - 9$

**Use the remainder theorem to find f(c).**

5. $f(x) = 3x^3 - x^2 + 5x - 4;\ c = 2$

$$\begin{array}{r|rrrr} 2 & 3 & -1 & 5 & -4 \\ & & 6 & 10 & 30 \\ \hline & 3 & 5 & 15 & 26 \end{array}$$

6. $f(x) = x^4 - 6x^2 + 4x - 8;\ c = -3$

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -6 & 4 & -8 \\ & & -3 & 9 & -9 & 15 \\ \hline & 1 & -3 & 3 & -5 & 7 \end{array}$$

**Use the factor theorem to show that x-c is a factor of f(x).**

7. $f(x) = x^3 + x^2 - 2x + 12;\ c = -3$

$$\begin{array}{r|rrrr} -3 & 1 & 1 & -2 & 12 \\ & & -3 & 6 & -12 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

$f(-3)=0$ so $x+3$ is a factor

8. $f(x) = x^{12} - 4096;\ c = -2$

$(-2)^{12} - 4096$

$4096 - 4096 = 0$

$f(-2)=0$ so $x+2$ is a factor

**Find a polynomial f(x) with leading coefficient 1 and having the given degree and zeros.**

9. degree 3; zeros -2, 0, 5

$f(x) = a(x+2)(x)(x-5)$

$f(x) = x^3 - 3x^2 - 10x$

10. degree 4; zeros -2, ±1, 4

$f(x) = a(x+2)(x-1)(x+1)(x-4)$

$f(x) = x^4 - 2x^3 - 9x^2 + 2x + 8$

**Use synthetic division to find the quotient and remainder if the first polynomial is divided by the second.**

11. $2x^3 - 3x^2 + 4x - 5;\ x-2$

$$\begin{array}{r|rrrr} 2 & 2 & -3 & 4 & -5 \\ & & 4 & 2 & 12 \\ \hline & 2 & 1 & 6 & 7 \end{array}$$

Q: $2x^2 + x + 6$

R: $7$

12. $x^3 - 8x - 5;\ x + 3$

$$\begin{array}{r|rrrr} -3 & 1 & 0 & -8 & -5 \\ & & -3 & 9 & -3 \\ \hline & 1 & -3 & 1 & -8 \end{array}$$

Q: $x^2 - 3x + 1$

R: $-8$

13. $3x^5 + 6x^2 + 7$; $x + 2$

$$\begin{array}{r|rrrrrr} -2 & 3 & 0 & 0 & 6 & 0 & 7 \\ & & -6 & 12 & -24 & 36 & -72 \\ \hline & 3 & -6 & 12 & -18 & 36 & -65 \end{array}$$

Q: $3x^4 - 6x^3 + 12x^2 - 18x + 36$

R: $-65$

14. $4x^4 - 5x^2 + 1$; $x - 1/2$

$$\begin{array}{r|rrrrr} \frac{1}{2} & 4 & 0 & -5 & 0 & 1 \\ & & 2 & 1 & -2 & -1 \\ \hline & 4 & 2 & -4 & -2 & 0 \end{array}$$

Q: $4x^3 + 2x^2 - 4x - 2$

R: $0$

**Use synthetic division to find f(c).**

15. $f(x) = 2x^3 + 3x^2 - 4x + 4$; $c = 3$

$$\begin{array}{r|rrrr} 3 & 2 & 3 & -4 & 4 \\ & & 6 & 27 & 69 \\ \hline & 2 & 9 & 23 & 73 \end{array}$$

$f(3) = 73$

16. $f(x) = -x^3 + 4x^2 + x$; $c = -2$

$$\begin{array}{r|rrrr} -2 & -1 & 4 & 1 & 0 \\ & & 2 & -12 & 22 \\ \hline & -1 & 6 & -11 & 22 \end{array}$$

$f(-2) = 22$

**Use synthetic division to show that c is a zero of f(x).**

17. $f(x) = 3x^4 + 8x^3 - 2x^2 - 10x + 4$; $c = -2$

$$\begin{array}{r|rrrrr} -2 & 3 & 8 & -2 & -10 & 4 \\ & & -6 & -4 & 12 & -4 \\ \hline & 3 & 2 & -6 & 2 & 0 \end{array}$$

$f(-2) = 0$

18. $f(x) = 4x^3 - 6x^2 + 8x - 3$; $c = 1/2$

$$\begin{array}{r|rrrr} 1/2 & 4 & -6 & 8 & -3 \\ & & 2 & -2 & 3 \\ \hline & 4 & -4 & 6 & 0 \end{array}$$

$f(\frac{1}{2}) = 0$

**Find all values of k such that f(x) is divisible by the given linear polynomial.**

19. $f(x) = kx^3 + x^2 + k^2x + 3k^2 + 11$; $x + 2$

$0 = (-2)^3k + (-2)^2 + (-2)k^2 + 3k^2 + 11$

$0 = -8k + 4 + k^2 + 11$

$0 = k^2 - 8k + 15$

$0 = (k - 3)(k - 5)$, $k = 3$, $k = 5$