

Find (a) $(f+g)(3)$

$$1. f(x) = x + 3; \quad g(x) = x^2$$

$$\begin{aligned} f(3) + g(3) &= 6 + 9 = 15 \\ f(3) - g(3) &= 6 - 9 = -3 \\ f(3) \cdot g(3) &= 6 \cdot 9 = 54 \\ f(3) / g(3) &= 6/9 = \frac{2}{3} \end{aligned}$$

(b) $(f-g)(3)$ (c) $(fg)(3)$ (d) $(f/g)(3)$

$$2. f(x) = -x^2; \quad g(x) = 2x - 1$$

$$\begin{aligned} f(3) + g(3) &= -9 + 5 = -4 \\ f(3) - g(3) &= -9 - 5 = -14 \\ f(3) \cdot g(3) &= -9 \cdot 5 = -45 \\ f(3) / g(3) &= -9/5 \end{aligned}$$

Find

(a) $(f+g)(x), (f-g)(x), (fg)(x), (f/g)(x)$ (b) the domain of $f+g, f-g, \text{ and } fg$ (c) the domain of f/g

(b) domain: All R's except 4, -5

$$3. f(x) = x^2 + 2, \quad g(x) = 2x^2 - 1$$

$$\begin{aligned} f(x) + g(x) &= (x^2 + 2) + (2x^2 - 1) \\ &= 3x^2 + 1 \\ f(x) - g(x) &= (x^2 + 2) - (2x^2 - 1) \\ &= 3 - x^2 \\ f(x) \cdot g(x) &= (x^2 + 2)(2x^2 - 1) \\ &= 2x^4 + 3x^2 - 2 \end{aligned}$$

$$\frac{f(x)}{g(x)} = \frac{x^2 + 2}{2x^2 - 1}; \quad \text{domain: All R's except } \pm \frac{\sqrt{2}}{2}$$

$$4. f(x) = \sqrt{x+5}, \quad g(x) = \sqrt{x+5}$$

$$\begin{aligned} f(x) + g(x) &= \sqrt{x+5} + \sqrt{x+5} \\ &= 2\sqrt{x+5} \\ f(x) - g(x) &= \sqrt{x+5} - \sqrt{x+5} \\ &= 0 \\ f(x) \cdot g(x) &= (\sqrt{x+5})(\sqrt{x+5}) \\ &= x+5 \end{aligned}$$

$$5. f(x) = \frac{2x}{x-4}, \quad g(x) = \frac{x}{x+5}$$

$$\begin{aligned} \frac{2x}{x-4} + \frac{x}{x+5} &= \frac{-2x(x+5)+x(x-4)}{(x-4)(x+5)} \\ &= \frac{3x^2+6x}{(x-4)(x+5)} \\ \frac{2x}{x-4} - \frac{x}{x+5} &= \frac{x^2+14x}{(x-4)(x+5)} \end{aligned}$$

Find

(a) $(f \circ g)(x)$ (b) $(g \circ f)(x)$ (c) $(f \circ f)(x)$ (d) $(g \circ g)(x)$

$$6. f(x) = 2x - 1, \quad g(x) = -x^2$$

$$f(g(x)) = f(-x^2) = 2(-x^2) - 1 = -2x^2 - 1$$

$$g(f(x)) = g(2x - 1) = -(2x - 1)^2 = -(4x^2 - 4x + 1) \\ = -4x^2 + 4x - 1$$

$$f(f(x)) = f(2x - 1) = 2(2x - 1) - 1 = 4x - 2 - 1 \\ = 4x - 3$$

$$g(g(x)) = g(-x^2) = -(-x^2)^2 = -x^4$$

$$7. f(x) = 3x^2, \quad g(x) = x - 1$$

$$f(g(x)) = f(x-1) = 3(x-1)^2 \quad \text{domain: All R's except } -5, 0, \frac{1}{4}$$

$$g(f(x)) = g(3x^2) = (3x^2) - 1 = 3x^2 - 1$$

$$f(f(x)) = f(3x^2) = 3(3x^2)^2 = 3(9x^4) = 27x^4$$

$$g(g(x)) = g(x-1) = x-1 - 1 = x-2$$

Find

(a) $(f \circ g)(x)$

(b) $(g \circ f)(x)$

(c) $f(g(-2))$

(d) $g(f(3))$

3. $f(x) = 2x - 5$

$g(x) = 3x + 7$

$$f(g(x)) = f(3x+7) = 2(3x+7) - 5 = 6x + 9$$

$$g(f(x)) = g(2x-5) = 3(2x-5) + 7 = 6x - 8$$

$$f(g(-2)) = f(3(-2)+7) = f(1) = 2(1) - 5 = -3$$

$$g(f(3)) = g(2(3)-5) = g(1) = 3(1) + 7 = 10$$

9. $f(x) = 3x^2 + 4$, $g(x) = 5x$

$$f(g(x)) = f(5x) = 3(5x)^2 + 4 = 75x^2 + 4$$

$$g(f(x)) = g(3x^2 + 4) = 5(3x^2 + 4) = 15x^2 + 20$$

$$f(g(-2)) = f(5 \cdot (-2)) = f(-10) = 3(-10)^2 + 4$$

$$g(f(3)) = g(3(3^2) + 4) = g(31) = 5(31)$$

$$= 155$$

10. $f(x) = 2x^2 + 3x - 4$, $g(x) = 2x - 1$

$$f(g(x)) = f(2x-1) = 2(2x-1)^2 + 3(2x-1) - 4 \\ = 8x^2 - 2x - 5$$

$$g(f(x)) = g(2x^2 + 3x - 4) = 2(2x^2 + 3x - 4) - 1 \\ = 4x^2 + 6x - 9$$

$$f(g(-2)) = f(2(-2)-1) = f(-5) = 2(-5)^2 + 3(-5) - 4$$

$$g(f(3)) = g(2(3)^2 + 3(3) - 4) = g(23) \\ = 2(23) - 1 = 45$$

11. $f(x) = 4x$, $g(x) = 2x^3 - 5x$

$$f(g(x)) = f(2x^3 - 5x) = 4(2x^3 - 5x) \\ = 8x^3 - 20x$$

$$g(f(x)) = g(4x) = 2(4x)^3 - 5(4x) \\ = 128x^3 - 20x$$

$$f(g(-2)) = f(2(-2)^3 - 5(-2)) = f(-6)$$

$$g(f(3)) = g(4(3)) = 2(12)^3 - 5(12) \\ = 3456 - 60 \\ = 3396$$

Solve the equation $(f \circ g)(x) = 0$

12. $f(x) = x^2 - 2$, $g(x) = x + 3$

$$f(g(x)) = f(x+3) = (x+3)^2 - 2 = 0 \\ = \sqrt{(x+3)^2} = \sqrt{2} \\ = x+3 = \pm\sqrt{2} \\ x = -3 \pm \sqrt{2}$$

13. $f(x) = x^2 - x - 2$, $g(x) = 2x - 1$

$$f(g(x)) = f(2x-1) = (2x-1)^2 - (2x-1) - 2 \\ = 4x^2 - 6x = 0 \\ 2x(2x-3) = 0 \\ x=0, x=\frac{3}{2}$$