*AAT

Chapter 3: 3-4 Definition of Function (IC/HW)-Days 1 & 2

Name: Key

Date: ____________ Period: ____________

1. **If f(x) = $-x^2 - x - 4$, find f(-2), f(0), and f(4).**

$f(-2) = -4 + 2 - 4 = -6$

$f(0) = 0 - 0 - 4 = -4$

$f(4) = -16 - 4 - 4 = -24$

2. **If f(x) = $\sqrt{x-4} - 3x$, find f(4), f(8), and f(13).**

$f(4) = \sqrt{4-4} - 3(4) = -12$

$f(8) = \sqrt{8-4} - 3(8) = -22$

$f(13) = \sqrt{13-4} - 3(13) = -36$

If a and h are real numbers, find

a) f(a)   (b) f(-a)   (c) -f(a)   (d) f(a+h)   (e) f(a) + f(h)   (f) $\frac{f(a+h)-f(a)}{h}$ $if\ h \neq 0$

3. f(x) = 5x - 2

$f(a) = 5(a) - 2 = 5a - 2$

$f(-a) = 5(-a) - 2 = -5a - 2$

$-f(a) = -1(5a - 2) = -5a + 2$

$f(a+h) = 5(a+h) - 2 = 5a + 5h - 2$

$f(a) + f(h) = 5a - 2 + 5h - 2 = 5a + 5h - 4$

$\frac{f(a+h) - f(a)}{h} = \frac{(5a+5h-2) - (5a-2)}{h} = \frac{5h}{h} = 5$

4. f(x) = $-x^2 + 4$

$f(a) = -(a)^2 + 4 = -a^2 + 4$

$f(-a) = -(-a)^2 + 4 = -a^2 + 4$

$-f(a) = -1(-a^2 + 4) = a^2 - 4$

$f(a+h) = -(a+h)^2 + 4 = -(a^2 + 2ah + h^2) + 4$
$= -a^2 - 2ah - h^2 + 4$

$f(a) + f(h) = (-a^2 + 4) + (-h^2 + 4) = -a^2 - h^2 + 8$

$\frac{f(a+h) - f(a)}{h} = \frac{(-a^2 - 2ah - h^2 + 4) - (-a^2 + 4)}{h} = \frac{-2ah - h^2}{h}$
$= -2a - h$

5. f(x) = $x^2 - x + 3$

$f(a) = a^2 - a + 3$

$f(-a) = (-a)^2 - (-a) + 3 = a^2 + a + 3$

$-f(a) = -1(a^2 - a + 3) = -a^2 + a - 3$

$f(a+h) = (a+h)^2 - (a+h) + 3$
$= a^2 + 2ah + h^2 - a - h + 3$

$f(a) + f(h) = (a^2 - a + 3) + (h^2 - h + 3) = a^2 + h^2 - a - h + 6$

$\frac{f(a+h) - f(a)}{h} = \frac{(a^2 + 2ah + h^2 - a - h + 3) - (a^2 - a + 3)}{h} = \frac{2ah + h^2 - h}{h}$
$= 2a + h - 1$

If a is a positive real number, find   (a) $g(\frac{1}{a})$ (b) $\frac{1}{g(a)}$ (c) $g(\sqrt{a})$ (d) $\sqrt{g(a)}$

6. g(x) = $4x^2$

$g(\frac{1}{a}) = 4(\frac{1}{a})^2 = \frac{4}{a^2}$

$\frac{1}{g(a)} = \frac{1}{4a^2}$

$g\sqrt{a} = 4(\sqrt{a})^2 = 4a$

$\sqrt{g(a)} = \sqrt{4a^2} = 2|a|$

7. g(x) = 2x - 5

$g(\frac{1}{a}) = 2(\frac{1}{a}) - 5 = \frac{2}{a} - 5$ or $\frac{2-5a}{a}$

$g(\sqrt{a}) = 2\sqrt{a} - 5$

$\frac{1}{g(a)} = \frac{1}{2(a) - 5} = \frac{1}{2a - 5}$

$\sqrt{g(a)} = \sqrt{2a - 5}$

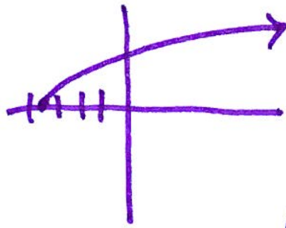
Find the domain of f.

8. f(x) = $\sqrt{2x+7}$

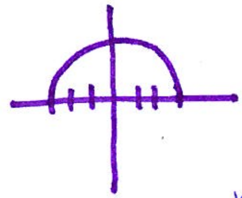
$2x+7 \ge 0$

$x \ge -\frac{7}{2}$

$[-\frac{7}{2}, \infty)$



9. f(x) = $\sqrt{9-x^2}$



$9-x^2 \ge 0$

$3 \ge |x|$

$-3 \le x \le 3$

$[-3, 3]$

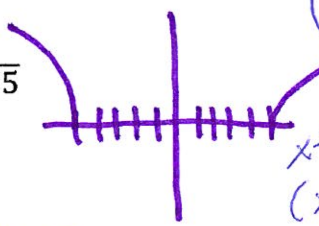
10. f(x) = $\frac{x+1}{x^3-4x}$

$x^3-4x=0$

$x(x+2)(x-2)=0$

All R's except 0, -2, 2

11. f(x) = $\sqrt{x^2-25}$



$x^2-25 \ge 0$

$|x| \ge 5$

$x \ge 5$ or $x \le -5$

$(-\infty, -5] \cup [5, \infty)$

12. $\frac{\sqrt{2x-3}}{x^2-5x+4}$

$x^2-5x+4=0$

$(x-4)(x-1)=0$

$x=1, 4$

$2x-3 \ge 0$

$x \ge \frac{3}{2}$

All R's $\ge \frac{3}{2}$ except 4

$[\frac{3}{2}, 4) \cup (4, \infty)$

13. $\frac{x-4}{\sqrt{x-2}}$

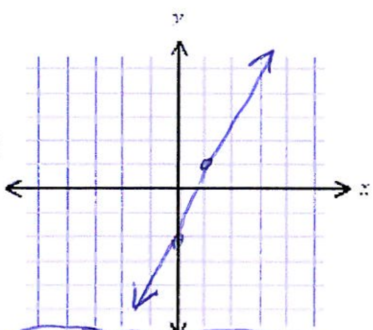
$x-2>0$

$x>2$

$(2, \infty)$

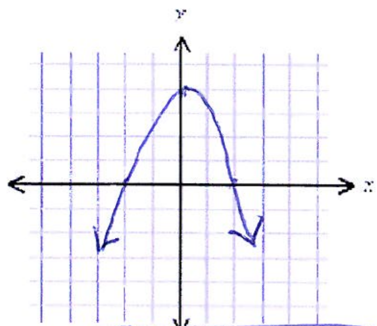
Sketch the graph of f. Find the domain D and range R of f.

14. f(x) = 3x - 2



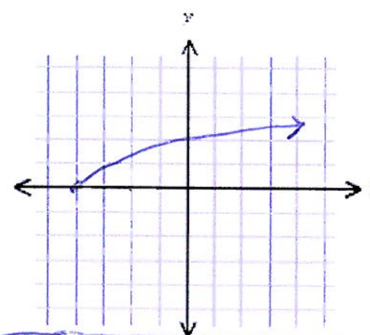
D: $(-\infty, \infty)$; R: $(-\infty, \infty)$

15. f(x) = 4 - x^2



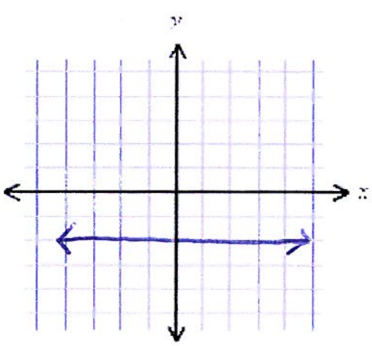
D: $(-\infty, \infty)$; R: $(-\infty, 4]$

16. f(x) = $\sqrt{x+4}$



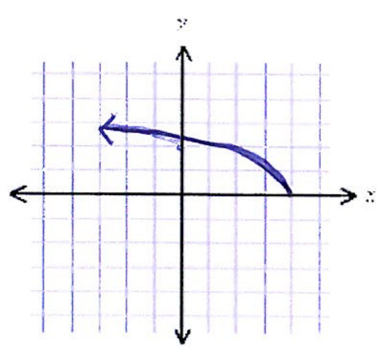
D: $[-4, \infty)$; R: $[0, \infty)$

17. f(x) = -2



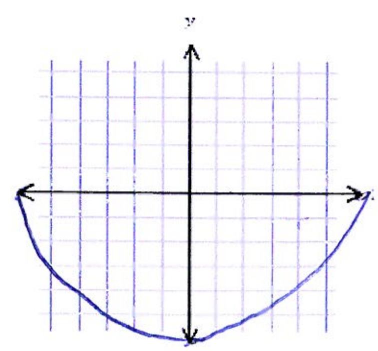
D: $(-\infty, \infty)$; R: $\{-2\}$

18. f(x) = $\sqrt{4-x}$



D: $(-\infty, 4]$; R: $[0, \infty)$

19. f(x) = $-\sqrt{36-x^2}$



D: $[-6, 6]$; R: $[-6, 0]$

If a linear function f satisfies the given conditions, find f(x).

20. f(-3) = 1 and f(3) = 2

$(-3,1),(3,2)$ $\quad m=\frac{2-1}{3-(-3)}=\frac{1}{6}$ $\quad y=\frac{1}{6}x+b$

$2=\frac{1}{6}(3)+b$

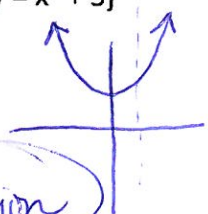
$\frac{3}{2}=b$

$y=\frac{1}{6}x+\frac{3}{2}$

Determine whether the set W of ordered pairs is a function.

21. W = {(x,y): $2y = x^2 + 5$}

$y=\frac{x^2+5}{2}$

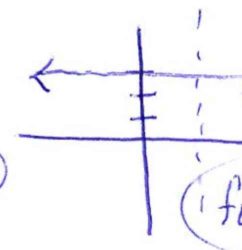


function

22. W = {(x,y): $x^2 + y^2 = 4$}

Not a function

23. W = {(x,y): y = 3}



function

24. W = {(x,y): xy = 0}

$\{(0,1),(0,2),(\ldots)\}$

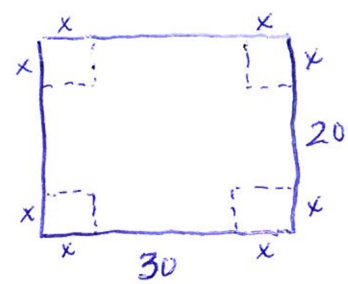
Not a function

25. W = {(x,y): |y| = |x|}

$\{(1,1),(1,-1),(\ldots)\}$

Not a function

26. From a rectangular piece of cardboard having dimensions 20 inches x 30 inches, an open box is to be made by cutting out an identical square of area x^2 from each corner and turning up the sides. Express the volume V of the box as a function of x.



$V=l\cdot w\cdot h$

$V(x)=(30-2x)(20-2x)(x)$ or $2(15-x)\cdot 2(10-x)(x)$

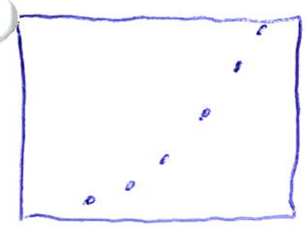
$V(x)=4x(15-x)(10-x)$

27. The table lists the practical stopping distances D (in feet) for cars at speeds S (in miles per hour) on level surfaces, as used by the American Association of State Highway and Transportation Officials.

S	20	30	40	50	60	70
D	33	86	167	278	414	593

[0,75,10] by [0,600,100]

(a) Plot the data.



(b) Determine whether stopping distance is a linear function of speed.

Not linear;

if linear, doubling speed would require ≈ 2x stopping distance

(c) Discuss the practical implications of these data for safely driving a car.

$\frac{414}{86}\approx 4.81$ if 2x speed, requires almost 5x stopping distance.